

BRIEF THEORY OF COHERENT PROCESSOR

INTRODUCTION

THE PARTICULAR COHERENT DATA PROCESSOR WHICH IS TO BE DISCUSSED RELIES HEAVILY UPON SEVERAL BASIC CONCEPTS. THESE CONCEPTS REPRESENT A DEPARTURE FROM CONVENTIONAL PRACTICE AND HENCE THE THEORETICAL DESCRIPTION MUST AWAIT THEIR PRESENTATION IN ORDER TO GAIN SOME CONTINUITY. WE WILL ACCORDINGLY PRESENT THE UNDERLYING ASSUMPTIONS AND SIGNAL PHYSICS PRIOR TO DESCRIPTION OF THE PROCESSOR ITSELF.

PHYSICAL CONCEPTS

1. SIGNAL ENERGY DENSITY

NO MATTER WHAT MEANS IS USED, ANY INTERCEPTION OF A SIGNAL WHICH IS ITSELF CAPABLE OF DOING WORK, IMPLIES AN ENERGY DENSITY ASSOCIATED WITH THAT SIGNAL. IF THE INTERCEPT IS THAT OF A DISTANT ACOUSTIC EVENT, THEN THE STATE OF THE MEDIUM AT THE POINT OF PICKUP IS SUCH AS TO HAVE A TOTAL ^{SIGNAL} ENERGY DENSITY ATTRIBUTABLE TO THAT DISTANT EVENT. THE TOTAL ENERGY DENSITY, $E(s)$, IS EXPRESSED IN JOULES PER CUBIC METER AND REPRESENTS THE AMOUNT OF WORK WHICH COULD BE PERFORMED AT ANY MOMENT IF THE AGENT OF PERCEPTION WERE SUFFICIENTLY CLEVER. FOR A NON-TURBULENT (VECTOR ONLY) MEDIUM IT CAN BE STATED THAT THE TOTAL ENERGY DENSITY, $E(s)$, IS PARTITIONED INTO A KINETIC ENERGY DENSITY, $T(s)$, AND A POTENTIAL ENERGY DENSITY, $V(s)$. FOR AN ACOUSTIC MEDIUM OF ENERGY PROPAGATION THE KINETIC ENERGY DENSITY IS PROPORTIONAL TO THE SQUARE OF PARTICLE VELOCITY WHILE THE POTENTIAL ENERGY DENSITY IS PROPORTIONAL TO THE SQUARE OF INSTANTANEOUS PRESSURE DEVIATION FROM EQUILIBRIUM.

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IT CAN BE PROVEN THAT THE FOLLOWING RELATIONSHIP EXISTS FOR ENERGY DENSITY PARTITIONING,

$$\sqrt{E(s)} = \sqrt{T(s)} + i \sqrt{V(s)} \quad (1)$$

WHERE THE TERMS $\sqrt{T(s)}$ AND $\sqrt{V(s)}$ ARE HILBERT TRANSFORMS OF EACH OTHER.

THE IMPORTANT CONSEQUENCE OF THIS IS THAT IF ONE ATTEMPTS A CATEGORIZATION OF A SIGNAL BASED SOLELY ON PRESSURE ONLY, OR VELOCITY ONLY MEASUREMENTS, HE HAS NOT UTILIZED ALL THE INFORMATION NECESSARY TO SPECIFY THAT SIGNAL'S TOTAL ENERGY DENSITY WITHOUT FURTHER MATHEMATICAL MANIPULATION. A FURTHER CONSEQUENCE OF THIS RELATION IS THAT BY UTILIZATION OF HILBERT TRANSFORMATION IT IS POSSIBLE TO OBTAIN TOTAL ENERGY DENSITY FROM A MEASUREMENT MADE OF ONE COMPONENT.

IN A SYSTEM THE ENERGY DENSITY REPRESENTING THE INSTANTANEOUS CAPABILITY TO DO WORK IS $E(t)$ AND HAS THE DIMENSIONS OF JOULES PER SECOND. IF THIS ENERGY IS CONVERTED TO WORK, THE TOTAL WORK PERFORMED ON THE TRANSDUCER IN A TIME T IS,

$$\mathcal{E} = \int_T dE = \int_T E(t) dt, \quad \text{JOULES} \quad (2)$$

2. CORRELATION AS A VECTOR OPERATION

THE TERM CORRELATION, AS UTILIZED IN CONVENTIONAL SIGNAL ANALYSIS, RELATES TO THAT SCALAR PROPERTY OBTAINED AS THE FOLLOWING INTEGRAL OF TWO SCALAR SIGNALS,

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} f(x) g(x+\tau) dx \quad (3)$$

ACCORDING TO THIS DEFINITION, THE CORRELATION BETWEEN A SINE WAVE AND A COSINE WAVE IS ZERO. A MAXIMUM VALUE OF CORRELATION WILL OCCUR WHEN AN IDENTITY EXISTS BETWEEN THE TWO SIGNAL FUNCTIONS.

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FOR THE PURPOSE OF OUR ANALYSIS AN ASSUMPTION IS MADE, BASED ON EQUATION (1). THAT A GENERAL SIGNAL MUST BE CONSIDERED A VECTOR AND NOT JUST A SCALAR. IT CAN BE SHOWN THAT A TIME DOMAIN REPRESENTATION OF A SIGNAL UNDER THESE GROUND RULES MAY BE GIVEN AS THE VECTOR, $h(t)$, WHERE,

$$h(t) = f(t) + i^{\circ} g(t) \quad (4)$$

THE SCALAR COMPONENTS $f(t)$ AND $g(t)$ ARE HILBERT TRANSFORMS OF EACH OTHER AND RELATE TO ENERGY DENSITY IN THE MANNER SHOWN IN EQUATION (1). THE VECTOR $h(t)$ HAS ALSO RECEIVED RECOGNITION IN NARROW BAND COMMUNICATION PROCESSES WHERE IT IS KNOWN AS THE ANALYTIC SIGNAL. FOR THIS ANALYSIS THE RELATION (4) IS CONSIDERED TRUE FOR ALL PROCESSES. THE RESULT OF THIS IS THAT A SIGNAL MUST ALWAYS BE TREATED AS A VECTOR FOR COMPLETE CHARACTERIZATION.

★ ANOTHER IMPORTANT RESULT RELATES TO THE CORRELATION BETWEEN TWO SIGNALS. IT MUST FOLLOW THAT CORRELATION MUST BE CONSIDERED TO BE A VECTOR, NOT JUST A SCALAR. FOR THE PURPOSE OF THIS ANALYSIS THE PROCESS OF EQUATION (3), UTILIZING VECTOR SIGNAL COMPONENTS, WILL BE CALLED CORRELATION. CORRELATION WILL THEN REFER TO THAT COMPLEX MEASURE OF SAMENESS BETWEEN TWO COMPLEX SIGNALS. IT IS ALSO OF SOME MERIT TO CONSIDER CORRELATION BETWEEN TWO SIGNALS AS REPRESENTING THAT OPERATION WHICH MUST BE PERFORMED ON ONE SIGNAL IN ORDER TO PRODUCE AN IDENTITY WITH ANOTHER SIGNAL WHEN EVALUATED BY THE INTEGRAL PROCESS OF (3). THUS THE CORRELATION BETWEEN A SINE WAVE AND A COSINE WAVE SIGNAL OBTAINED FROM A LIKE ENERGY RELATED PROCESS, SUCH AS PRESSURE RESPONSE, IS THE VECTOR $i^{\circ} = \sqrt{-1}$.

3. SIGNAL

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FOR THE PURPOSE OF DISCUSSION IT WILL BE ASSUMED THAT THE SIGNAL

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OF INTEREST IS A SINGLE FREQUENCY SINUSOID OF VOLTAGE TIME DEPENDENCE $v(t)$.

IN TERMS OF THE ENERGY WHICH IS AVAILABLE TO DO WORK WHEN THIS VOLTAGE IS IMPRESSED ACROSS A UNIT RESISTOR,

$$V(t) = \sqrt{2E(t)} \sin \omega t, \quad \text{JOULE}^{\frac{1}{2}} - \text{SEC}^{-\frac{1}{2}} \quad (5)$$

A GENERAL SIGNAL OF UNIT VOLTAGE DEPENDENCE $s(t)$ MAY BE EXPRESSED AS,

$$V(t) = \sqrt{E(t)} \cdot s(t), \quad \text{JOULE}^{\frac{1}{2}} - \text{SEC}^{-\frac{1}{2}} \quad (6)$$

THE POWER, OR RATE OF CHANGE OF ENERGY, MADE AVAILABLE WHEN THE VOLTAGE IS IMPRESSED ACROSS A UNIT RESISTOR IS,

$$P_s = v^2(t), \quad \text{JOULE} - \text{SEC}^{-1} = \text{WATT} \quad (7)$$

4. NOISE

FOR CONVENIENCE IT WILL BE ASSUMED THAT THE NOISE IS A ZERO MEAN GAUSSIAN WHITE RANDOM VARIABLE OF VARIANCE σ^2 AND WITH A DOUBLE-SIDED ENERGY SPECTRAL DENSITY OF $N_0/2$ JOULES PER HERTZ. THIS MEANS THAT A PERFECT FILTER WHICH PASSED ONLY FREQUENCIES FROM f TO $(f+1)$ HERTZ WHEN MEASURED BY CONVENTIONAL OSCILLATORS AND VOLTMETERS WOULD HAVE AN OUTPUT ENERGY OF N_0 JOULES. A TRUE RMS METER WOULD, IN THIS INSTANCE, MEASURE $\sqrt{N_0}$ VOLTS ACROSS A UNIT RESISTOR UNDER CONDITIONS OF MAXIMUM ENERGY TRANSFER.

5. SIGNAL TO NOISE

THE SIGNAL TO NOISE RATIO, α , WILL BE DEFINED AS THE AVERAGE SIGNAL POWER TO AVERAGE NOISE POWER IN A ONE HERTZ BANDWIDTH CENTERED ON THE FREQUENCY OF THE SIGNAL WHEN BOTH SIGNAL AND NOISE IS IMPRESSED ON THE SAME IMPEDANCE. THE ASSUMED DEFINITION OF SIGNAL AND NOISE ALLOWS THIS TO BE EXPRESSED ANALYTICALLY AS,

$$\alpha = \frac{E}{N_0}, \quad \text{WATT/WATT} \quad (8)$$

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BOTH SIGNAL AND NOISE HAVE DOUBLE SIDED SPECTRAL DEPENDENCE. THE ONE HERTZ BANDWIDTH REFERRED TO IS THAT WHICH WOULD PASS ONLY FREQUENCIES FROM f HERTZ TO $(f + 1)$ HERTZ WHEN MEASURED BY CONVENTIONAL OSCILLATORS AND VOLTMETERS.

6. MATCHED FILTER

IF A SIGNAL BE SUCH THAT IT MAY BE EXPRESSED AS $s(t)$ THEN A FILTER THROUGH WHICH $s(t)$ IS TO BE PASSED WILL BE DEFINED AS MATCHED TO $s(t)$ FOR A TIME T IF THE IMPULSE RESPONSE OF THE FILTER IS THE COMPLEX CONJUGATE OF THE TIME REVERSED SIGNAL,

$$h(t) = s^*(T-t) \quad (9)$$

THE OUTPUT POWER AVAILABLE TO A UNIT LOAD FROM A FILTER MATCHED TO A SIGNAL OF ENERGY E JOULES IS, AT THE END OF T SECONDS, *

$$P_S = E^2, \quad \text{WATT} \quad (10)$$

THE OUTPUT POWER AVAILABLE TO A UNIT LOAD FROM A FILTER MATCHED TO A SIGNAL OF ENERGY E BUT FED FROM A NOISE OF DOUBLE SIDED ENERGY DENSITY $N_0/2$ JOULES PER HERTZ IS,

$$P_N = \frac{N_0 \cdot E}{T}, \quad \text{WATT} \quad (11)$$

THE OUTPUT SIGNAL TO NOISE RATIO AVAILABLE FROM A FILTER MATCHED TO A SIGNAL OF ENERGY E FOR A TIME T BUT FED BOTH THE ABOVE SIGNAL AND NOISE IS,

$$\alpha_0 = \frac{P_S}{P_N} = \frac{E \cdot T}{N_0} = \alpha T, \quad \text{WATT/WATT} \quad (12)$$

ENERGY PLANE PORTRAYAL

THE INFORMATION OF INTEREST IS THE TOTAL SIGNAL ENERGY DENSITY AS A FUNCTION OF TIME AND/OR FREQUENCY. THERE IS A VECTOR RELATIONSHIP BETWEEN THAT SIGNAL COMPONENT REPRESENTING TOTAL ENERGY DENSITY AND THE SIGNAL COMPONENTS REPRESENTING KINETIC AND POTENTIAL ENERGY DENSITIES. CONSEQUENTLY A VERY USEFUL PORTRAYAL OF A SIGNAL INVOLVES A TWO DIMENSIONAL COMPLEX PLANE WITH ONE AXIS CORRESPONDING TO KINETIC ENERGY RELATED VALUES AND THE QUADRATURE AXIS CORRESPONDING TO POTENTIAL ENERGY RELATED VALUES. IN THIS SECTION WE SHALL DISCUSS SIGNAL TRAJECTORIES ON THIS COMPLEX ENERGY PLANE WHEN PROCESSED THROUGH A MATCHED FILTER. THE STATISTICAL DECISION CONCERNING SIGNAL CHARACTERIZATION WILL BE MADE WITHIN THE REFERENCE OF THIS ENERGY PLANE.

1. SIGNAL ONLY TRAJECTORY

ASSUME THAT THE OUTPUT OF A MATCHED FILTER IS PLOTTED AS A FUNCTION OF TIME FOR A SIGNAL WHICH STARTS AT TIME ZERO. THIS OUTPUT IS THAT OF THE COMPLEX CORRELATION INTEGRAL OF EQUATION (3). THE SIGNAL TRAJECTORY IS AS SHOWN IN FIGURE 1, CURVE 1.

BECAUSE WE PRESUME TO KNOW THE COMPLEX ENERGY PARTITIONING, THE TRAJECTORY COMMENCES AT TIME ZERO FROM THE ORIGIN AND UNIFORMLY TRAVELS THROUGH POINTS 2 AND 3 AND AT THE END OF T SECONDS RESIDES AT POINT 4. IF WE DID NOT KNOW THE PRECISE ENERGY PARTITIONING THE TRAJECTORY MAY BE THAT OF CURVE 2. IN THE CASE OF CURVE 1 THE CORRELATION COEFFICIENT WILL BE UNITY AT AN ANGLE ZERO DEGREES. IN THE CASE OF CURVE 2, THE CORRELATION COEFFICIENT WILL BE UNITY AT AN ANGLE θ DEGREES.

IF THE FILTER IS MATCHED TO A SIGNAL SLIGHTLY DIFFERENT IN FREQUENCY FROM THE ACTUAL SIGNAL, THE TRAJECTORY WILL BE AS SHOWN IN FIGURE 2.

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IN THIS CASE THE TRAJECTORY WILL BE A SEGMENT OF A CIRCLE COMMENCING AT THE ORIGIN EMERGING WITH SOME ANGLE θ_1 , REPRESENTING THE DIFFERENCE BETWEEN ASSUMED AND ACTUAL KINETIC ENERGY, AND DESCRIBE A CIRCULAR ARC TERMINATING AT AN ANGULAR SLOPE θ_2 . THE CORRELATION COEFFICIENT IS THE VECTOR $\rho e^{i\phi}$. THE FREQUENCY OFFSET IN HERTZ BETWEEN THE ACTUAL AND ASSUMED SIGNAL IS

$$\Delta f = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{2\pi T} \quad (13)$$

2. NOISE ONLY TRAJECTORY

FOR NOISE ONLY WE HAVE A SIGNAL $n(t)$ OF ZERO MEAN GAUSSIAN CHARACTERISTIC. THIS NOISE, JUST AS ANY OTHER SIGNAL, HAS A KINETIC AND POTENTIAL ENERGY RELATED COMPONENT. THE FILTER ASSUMED FOR THIS PROCESS IS MATCHED TO THE SIGNAL,

$$X(t) = S(t) \cdot \sqrt{E(t)} \quad (14)$$

FOR A TIME T . THE OUTPUT $y(t)$, FOR A SIGNAL $X(t)$ IS,

$$y(t) = \frac{1}{T} \int_{-\infty}^{\infty} X(\tau) h(t-\tau) d\tau \quad (15)$$

IN THE CASE OF THE SIGNAL ONLY, THE OUTPUT AT THE END OF T SECONDS IS,

$$y_s(t) = \frac{1}{T} \int_0^T S(\tau) E^{\frac{1}{2}}(\tau) S^*(\tau) E^{\frac{1}{2}}(\tau) d\tau = S^2(t) \cdot E \quad (16)$$

WHICH, BECAUSE $S(t)$ IS BY DEFINITION OF UNIT MAGNITUDE, BECOMES,

$$y_s(t) = E \quad (17)$$

THIS IS THE LENGTH OF THE VECTOR IN FIGURE 1.

FOR NOISE ONLY, $X(t)$ IS A ZERO MEAN GAUSSIAN AND SO,

$$\overline{y_N(t)} = 0 \quad (18)$$

BY DEFINITION.

THE SEPARATE ENERGY COMPONENTS BECOME,

$$\begin{aligned} \overline{a^2} &= \overline{b^2} = \overline{y_N^2(T)} \\ &= \frac{1}{T^2} \int_0^T \int_0^T \overline{n(\lambda) n^*(r) S(\lambda) E^{\frac{1}{2}}(\lambda) S^*(r) E^{\frac{1}{2}}(r)} d\lambda dr \quad (19) \end{aligned}$$

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BECAUSE OF THE ASSUMED DEFINITION FOR THE NOISE,

$$\overline{n(\lambda)n^*(r)} = \frac{N_0}{2} \delta(\lambda-r) \quad (20)$$

IT THEN FOLLOWS THAT THE ENERGY COORDINATES REACHED AT THE END OF T SECONDS, SHOWN IN FIGURE 3, IS,

$$\begin{aligned} \overline{a^2} = \overline{b^2} &= \overline{y_N^2(T)} \\ &= \frac{1}{T^2} \int_0^T \frac{N_0}{2} S^2(\lambda) E(\lambda) d\lambda = \frac{N_0 \cdot E}{2T} \end{aligned} \quad (21)$$

THE ENERGY PLANE TRAJECTORY IS A TWO DIMENSIONAL RANDOM WALK FROM THE ORIGIN TO THE TERMINAL POINT OF EQUATION (21). THE TOTAL MEAN SQUARE DISTANCE FROM THE ORIGIN, WHICH RELATES TO TOTAL ENERGY DENSITY, IS,

$$\sigma^2 = c^2 = a^2 + b^2 = \frac{2N_0 E}{2T} = \frac{N_0 E}{T} \quad (22)$$

THE PARAMETER c^2 IS THE VARIANCE OF THE DISTANCE FROM THE ORIGIN FOR SAMPLES STARTING FROM THE ORIGIN AND TERMINATING T SECONDS AFTER INITIATION.

3. SIGNAL PLUS NOISE

THE NOISE, BY DEFINITION, HAS A ZERO MEAN. THE STATISTICAL PROBABILITY THAT A NOISE ONLY VECTOR WILL BE OF LENGTH r FROM THE ORIGIN AT THE END OF T SECONDS IS THE WELL KNOWN RAYLEIGH BIVARIATE,

$$p(r) dr = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} dr \quad (23)$$

IT IS APPARENT THAT THE EFFECT OF ADDING A SIGNAL WILL BE THAT OF CREATING A MEAN TO THE DISTRIBUTION SINCE THE ENERGY PLANE RELATES TO MATCHED FILTER OUTPUT. THIS IS SKETCHED IN FIGURE 4.

IN TERMS OF THE DEFINITION PREVIOUSLY ASSUMED, THE PROBABILITY DENSITY FUNCTION (PDF) FOR THE DISTANCE FROM THE ORIGIN FOR SIGNAL PLUS NOISE IS,

$$p(r) = \frac{r}{\sigma^2} e^{-\left(\frac{r^2 + 2\sigma^2 \alpha}{2\sigma^2}\right)} I_0\left(\frac{r}{\sigma} \sqrt{2\alpha}\right) \quad (24)$$

THE RELATION OF EQUATION (24) IS SHOWN PLOTTED IN FIGURE 5 FOR NOISE ONLY AND SEVERAL VALUES OF SIGNAL TO NOISE RATIO. THE ASSUMED TIME OF INTEGRATION IS ONE SECOND, WHICH UNDER THE RELATION OF EQUATION (12) MAKES INPUT AND OUTPUT SIGNAL TO NOISE RATIOS EQUAL FOR THE ENERGY PLANE. FOR A T SECOND MATCHED FILTER THE NUMERICAL VALUE OF THE SIGNAL TO NOISE RATIO IN EQUATION (24) AND THOSE FOLLOWING WILL BE INCREASED T TIMES IN ACCORDANCE WITH EQUATION (12). THUS, IF THE INCOMING SIGNAL HAS A MEASURED α , IN ACCORDANCE WITH EQUATION (8), OF 0 dB THEN THE VALUE TO USE FOR A MATCHED FILTER OF 2 SECONDS LENGTH WILL BE +3 dB. FOR A FILTER OF 4 SECONDS LENGTH THE α WILL BE +6 dB, AND SO ON.

THE CURVES OF FIGURE 5 ARE THE PROBABILITY DENSITY FUNCTION (PDF) FOR THE VALUE OF THE VECTOR LENGTH FROM THE ORIGIN OF THE ENERGY PLANE AT THE END OF T SECONDS OF FILTER PROCESSING. THESE RELATE TO THE WAGERING ODDS ONE COULD USE TO MAKE AN ESTIMATE OF THE PROBABLE VECTOR LENGTH FOR ANY GIVEN SAMPLE. THE INTEGRAL OF THIS FUNCTION TO A POINT r , THE CUMULATIVE DISTRIBUTION, YIELDS THE PROBABILITY THAT ANY VECTOR LENGTH IS LESS THAN r .

WE OF COURSE ARE INTERESTED IN A VECTOR, NOT JUST A SCALAR LENGTH, SO THAT IT IS OF IMPORTANCE TO KNOW THE PDF FOR ANGLE. REFERRING TO FIGURE 4, ASSUME THAT THE MEANS OF THE DISTRIBUTION IS AT THE ANGLE θ . THE PDF FOR ANGULAR DISTRIBUTION AROUND THE DEFINED VALUE $\theta = 0$ IS,

$$p(\theta) = \frac{e^{-\alpha}}{2\pi} + \frac{1}{2} \sqrt{\frac{\alpha}{\pi}} \cos \theta \cdot e^{-\alpha \sin^2 \theta} [1 + \operatorname{erf}(\sqrt{\alpha} \cos \theta)] \quad (25)$$

AS CAN BE SEEN, THIS EQUATION IS MUCH MORE COMPLEX THAN THE SIMPLE RAYLEIGH DISTRIBUTION OF (24), AND CONSEQUENTLY IS DELETED FROM MOST FUNDAMENTAL TEXTS ON THE SUBJECT. UNFORTUNATELY IT IS ALSO DELETED

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AS AN EQUATION FROM MANY MORE ADVANCED TEXTS WITH THE CONSEQUENCE THAT MANY PEOPLE ARE UNAWARE OF THE GREATER SENSITIVITY OF THE ANGULAR PDF TO LOW VALUES OF α THAN THE VECTOR LENGTH PDF. WHEN ONE TALKS OF THE TOTAL POWER, SUCH AS IN (10), HE THROWS AWAY THIS ANGULAR PDF IN FAVOR OF (24). EQUATION (25) IS PLOTTED IN FIGURE 6.

IN FIGURE 7 ONE CAN SEE PICTORIALY THE INTERPLAY BETWEEN THE ANGULAR AND VECTOR LENGTH PDF. THIS PICTORIAL DISPLAY SHOWS THE *ICE CREAM CONE* LOCUS OF THOSE VECTORS WITH A GIVEN CUMULATIVE DISTRIBUTION. FOR EXAMPLE WITH A GIVEN α IT MAY BE STATED THAT X PERCENT OF ALL VECTORS MUST LIE WITHIN THE CIRCULAR PORTION OF THE ICE CREAM CONE.

DESCRIPTION OF COHERENT PROCESSOR

THE PREVIOUS VERY CURSORY PRESENTATION OF THE PHYSICAL CONCEPTS ALLOW FOR A DISCUSSION AT THIS POINT OF THE INSTRUMENTATION THAT IS THE SUBJECT OF THIS REPORT. THIS DESCRIPTION WILL GO ONLY TO THE DETAIL OF A BLOCK DIAGRAM AT THIS POINT. TWO MAJOR MODES WILL BE DESCRIBED. THESE RELATE TO SIGNAL DETECTION AND SIGNAL TRACKING.

SIGNAL DETECTION

FROM THE FOREGOING DISCUSSION OF PHYSICAL CONCEPTS IT IS OBVIOUS THAT THE PROCESS OF DETECTION OF A COHERENT SIGNAL IS REDUCIBLE TO THAT OF STATISTICAL DETERMINATION OF A MEAN TO THE DISTRIBUTION OF VECTORS ON THE APPROPRIATE ENERGY PLANE. THREE BASIC SUBSETS OF THIS DETECTION PROBLEM ARISE WHEN CONSIDERING THAT CLASS OF SIGNAL ASSUMED. WE MAY KNOW (A) BOTH FREQUENCY AND PHASE EXACTLY, (B) FREQUENCY ONLY EXACTLY, OR (C) FREQUENCY ONLY APPROXIMATELY. THROUGHOUT IT WILL BE ASSUMED THAT NO KNOWLEDGE EXISTS OF THE EXACT SIGNAL TO NOISE RATIO OR WHETHER THERE IS A SIGNAL AT ALL.

A) KNOWN FREQUENCY AND PHASE

ASSUME THAT KNOWING THE FREQUENCY AND PHASE, THE ENERGY PLANE DECISION FOR DETERMINATION OF A MEAN IS MADE BY EVALUATION OF SUCCESSIVE VECTORS AND DETERMINING THE NUMBER OF VECTORS IN THE HALF PLANE CONTAINING THE MEAN. THIS IS SHOWN IN FIGURE 8 WITH THE DECISION CRITERION THE NUMBER OF HITS WITHIN THE ANGLE $|\phi|$.

FIGURE 9 SHOWS THE CUMULATIVE DISTRIBUTION FOR ANGLES FROM THE MEAN AS A FUNCTION OF SEVERAL SIGNAL TO NOISE RATIOS. AS AN EXAMPLE OF THE USE OF THIS CURVE ASSUME AN α OF -6 dB. FIFTY NINE PERCENT OF THE

VECTORS WILL LIE WITHIN SIXTY DEGREES OF THE KNOWN MEAN ANGLE, AND SEVENTY FIVE PERCENT WILL LIE WITHIN NINETY DEGREES. IT IS A REMARKABLE FEATURE OF EQUATION (25) THAT THE CUMULATIVE DISTRIBUTION FOR ANGLES WITHIN NINETY DEGREES SIMPLIFIES TO THE EXPRESSION,

$$\int_{-\pi/2}^{\pi/2} p(\theta) d\theta = \frac{1}{2} \operatorname{erf} \sqrt{\alpha} \quad (26)$$

THIS IS PLOTTED IN FIGURE 10 AS A FUNCTION OF α .

FIGURE 11 SHOWS THE EQUIPMENT CAPABLE OF PRESENTING AN OSCILLOSCOPE DISPLAY OF THE ENERGY PLANE OF FIGURE 8. THE INCOMING SIGNAL IS SPLIT INTO TWO IDENTICAL CHANNELS. IN ONE CHANNEL IT IS MULTIPLIED BY ITS COMPLEX CONJUGATE (ASSUMED TO BE A SINE WAVE) AND IN THE OTHER CHANNEL BY THE HILBERT TRANSFORM OF ITS COMPLEX CONJUGATE (ASSUMED TO BE A COSINE WAVE). THE OUTPUT OF EACH MULTIPLIER FEEDS ITS OWN INTEGRATOR AND IN TURN SENT TO AN OSCILLOSCOPE. PRIOR TO SIGNAL EVALUATION S_x AND S_y ARE CLOSED INITIALIZING THE INTEGRATORS AT ZERO AND CENTERING THE OSCILLOSCOPE PLOT. AT TIME ZERO THE SWITCHES S_x AND S_y ARE OPENED ALLOWING INTEGRATION TO PROCEED AND THE CRT SPOT WILL COMMENCE ITS TWO DIMENSIONAL RANDOM WALK PLUS DRIFT FROM THE CENTER OF THE SCREEN. AT THE END OF T SECONDS THE SPOT WILL BE AT THE POSITION OF THE SIGNAL VECTOR UPON WHICH THE ANALYSIS WILL BE BASED. ASSUME NOW THAT THE CIRCUITRY OF FIGURE 12 IS CONNECTED TO THAT OF FIGURE 11.

THE ANGULAR DEPENDENCE, $\sin \theta$ AND $\cos \theta$, IS STRIPPED OUT AS SHOWN ALONG WITH THE VECTOR LENGTH ρ . A SIMPLE COORDINATE ROTATION MAY BE MADE FROM THE ASSUMED θ COORDINATE SYSTEM TO A ϕ COORDINATE SYSTEM DEFINED SUCH THAT $\phi = 0$ LIES ALONG THE KNOWN SIGNAL-ONLY TRAJECTORY IN THE ENERGY PLANE. A SIMPLE DETERMINATION OF HALF PLANE STATISTICS

MAY THEN BE MADE BY COUNTING A *HIT* AS A VECTOR WITH POSITIVE COSINE ϕ , AND AVERAGING HITS TO TOTAL POSSIBLE COUNTS FOR A RUNNING SAMPLE WHERE S_x AND S_y ARE SET TO ZERO AFTER EACH TALLY THEN OPENED FOR THE NEXT SAMPLE. AN EXAMPLE OF CIRCUITRY CAPABLE OF DOING THIS IS SHOWN IN FIGURE 13.

SYMBOLICALLY THE VECTOR ROTATION IS MADE BY TWO MULTIPLIERS AND A SUMMER. THE DECISION TO CLOCK A UNIT STATE INTO A SHIFT REGISTER IS MADE ON THE BASIS OF THE ALGEBRAIC SIGN OF THIS ROTATED VECTOR. IF, FOR EXAMPLE, THE SHIFT REGISTER IS 100 STAGES IN LENGTH THEN A 100 PERCENT *HIT* WILL FILL ALL STAGES WITH ONES. A 75 PERCENT *HIT* RATIO WILL HAVE THE SAME PERCENTAGE OF ONES. BY SIMPLY SUMMING A UNIT CURRENT FROM EACH STAGE WHEN IT CONTAINS A ONE AND NO CURRENT WITH A ZERO, ONE NEED ONLY AVERAGE THE 100 STAGES IN ORDER TO OBTAIN AN ANALOG INDICATION OF *HITS* TO *MISSES*, OR IF NEEDED A DIGITAL UPDATE CAN BE MAINTAINED BY CONVENTIONAL MEANS.

B) KNOWN FREQUENCY ONLY

IN THE CASE WHERE PHASE IS UNKNOWN, ONE HAS TWO ALTERNATIVES: HE MAY ELECT TO BASE A STATISTICAL SIGNAL ESTIMATE ON THE VECTOR LENGTH, OR HE MAY ACCUMULATE ENOUGH SAMPLES TO ESTIMATE PROBABLE PHASE AND ROTATE HIS PLANE OF REFERENCE IN ORDER TO USE THE STATISTICS OF KNOWN FREQUENCY AND PHASE. IN EITHER CASE SOME MEANS MUST BE FOUND TO ELIMINATE THE REQUIREMENT FOR INTIMATE KNOWLEDGE OF THE NOISE POWER. THIS MAY READILY BE DONE BY UTILIZING A VECTOR DECISION BASED ON UNIT AMPLITUDE VECTORS OF THE TYPE FED INTO FIGURE 13. THIS IS BECAUSE THE ANGLE STATISTICS WILL BE GAUSSIAN IF THE NOISE STATISTICS IS GAUSSIAN, AND WILL HAVE A MEAN INDICATIVE OF THE EXISTENCE OF A SIGNAL.

IN THIS CASE THE PDF IS STILL THE MOST POWERFUL INDICATOR WITH A DECISION MATRIX MADE MORE DIFFICULT SINCE THE EXACT PHASE IS NOT KNOWN. THREE COURSES OF ACTION ARE AVAILABLE.

1) SINCE THE PURPOSE OF ANALYSIS IS DETECTION OF A FIXED, KNOWN FREQUENCY SIGNAL THIS IS EQUIVALENT TO DETERMINING THE EXISTENCE OF A MEAN TO THE ANGLE PDF. THIS MAY BE DONE BY EVALUATING A SLIDING ACCUMULATION OF 180° SEGMENTS OF THE PDF. THIS IS SHOWN IN FIGURE 14 AS AN EVALUATION OF THE AREA UNDER SEGMENT 1, THEN UNDER SEGMENT 2 AND SO ON, CONSTANTLY EVALUATING THE RATIO OF THE RESPECTIVE AREAS TO UNITY. THIS IS BECAUSE A FULL 360° SEGMENT MUST BE CONSIDERED TO HAVE A UNIT PROBABILITY OF A *HIT*. A PLOT OF THE 180° AREAS VERSUS THE CENTRAL ANGLE OF EACH AREA SHOULD YIELD A PLOT SUCH AS SHOWN IN FIGURE 15. A SIMPLE DETERMINATION MAY THEN BE MADE OF THE PROBABILITY BY NOTING THE PEAK ON FIGURE 15 AND COMPARING WITH FIGURE 10.

2) A COMPLEX HARMONIC ANALYSIS OF THE ANGLE PDF SHOULD DISCLOSE THE EXISTENCE OF A FUNDAMENTAL AND HIGHER HARMONIC TERMS.

3) THE PDF ANGLES MAY HAVE A MEAN DISTRIBUTION AT ANY ANGLE RELATIVE TO LOCAL REFERENCE WHEN A SIGNAL IS PRESENT. A PDF OF DIFFERENCE ANGLES BETWEEN SUCCESSIVE SAMPLES, ON THE OTHER HAND, WILL ALWAYS HAVE ITS MEAN AT ZERO DEGREES. THIS DIFFERENCE ANGLE IS THAT OF $\Delta\theta$ IN FIGURE 16 WHERE A FIRST AND SECOND SAMPLE HAD ϕ_1 AND ϕ_2 RESPECTIVELY.

THE HALF PLANE STATISTICS OF $\Delta\theta$ IS GIVEN BY THE EXPRESSION,

$$\int_{-\pi/2}^{\pi/2} p(\Delta\theta) d(\Delta\theta) = \frac{1}{2} e^{-\alpha} \quad (27)$$

THIS IS SHOWN PLOTTED IN FIGURE 10.

c) FREQUENCY ONLY APPROXIMATELY

IF ONE DOES NOT KNOW THE EXACT FREQUENCY, THE ANGLE PDF WILL DETERIORATE VERY RAPIDLY. THE ONLY RECOURSE FOR EXTENDED ANALYSIS WITH A FIXED FREQUENCY LOCAL REFERENCE UTILIZING A SINGLE CHANNEL MATCHED FILTER WILL BE DIFFERENCE STATISTICS. ONE MUST THEN CHOOSE THE MATCHED PERIOD OF EACH SAMPLE, T , SUCH THAT THE PROBABLE SIGNAL TRAJECTORY DOES NOT ACCUMULATE MORE THAN PERHAPS A 90° SHIFT IN THIS PERIOD. FIGURE 17 SHOWS THE ON-FREQUENCY PDF OF DIFFERENCE ANGLE $\Delta\theta$. IF THE OFFSET BETWEEN INCOMING AND REFERENCE FREQUENCY IS SUCH AS TO ACCUMULATE NOT MORE THAN 45° IN THE PERIOD T ; THEN TO FIRST ORDER THE STATISTICS OF FIGURE 17 WILL HOLD AND ONE MAY ASSUME THE PDF OF $\Delta\theta$ IS SHIFTED BY JUST THIS OFFSET ANGLE. TO GIVE SOME COMPARISON ASSUME THAT THE OFFSET BETWEEN INCOMING SIGNAL AND REFERENCE IS 0.1 Hz AND ONE SECOND SAMPLES ARE ACCUMULATED. WITHIN TEN SAMPLES THE ANGLE STATISTICS WOULD HAVE TRAVERSED 360° AND BE USELESS. EACH DIFFERENCE ANGLE VECTOR WILL ON THE AVERAGE HAVE A SHIFT OF 36° ALLOWING AN INDEFINITE ACCUMULATION OF ONE SECOND SAMPLES FOR DEVELOPING AN ESTIMATOR OF THE PDF AND A DETERMINATION OF THIS MEAN.

THERE IS ANOTHER MEANS AVAILABLE FOR SIGNAL DETERMINATION IF EACH VECTOR CALCULATION IS MAINTAINED IN MEMORY AND TIME-TAGGED. IF THE DIFFERENCE STATISTIC, WHICH IS A RUNNING CALCULATION, SHOWS THE PROBABILITY OF A SIGNAL SLIGHTLY OFF FREQUENCY THEN ONE MAY BRING THE MORE POWERFUL ANGLE STATISTICS TO BEAR BY DRAWING FROM MEMORY EACH VECTOR AND ROTATING BY THE PROPER ANGLE TO BRING EACH INTO THE SAME PHASE ALIGNMENT FOR THE EXISTING EPOCH OF TIME. AS IN OUR EXAMPLE IF A SUFFICIENT SUSPICION WERE TO EXIST THAT A SIGNAL WAS PRESENT AND HAD A PROBABLE FREQUENCY.

OFFSET OF 0.1 Hz, ONE WOULD DRAW THE LATEST VECTOR FROM MEMORY AND ROTATE 36° , THE EARLIER VECTOR BY 72° , THE NEXT EARLIER VECTOR BY 108° , ETC. EACH PROPERLY ROTATED VECTOR WOULD THEN BE USED TO EVALUATE AN ANGLE PDF. IT IS OBVIOUS BY THIS MEANS THAT NOT ONLY CAN ONE RETAIN THE SIGNAL IMMEDIATE HISTORY SO AS TO CORRECT FOR OFFSET FREQUENCY, BUT BY USING A DICTIONARY OF FREQUENCY MANEUVER CORRECTIONS DETERMINE SIGNAL FREQUENCY BALLISTICS A POSTERIORI.

SIGNAL TRACKING

THE PROCESS OF TRACKING IN THIS REPORT MEANS THE ADAPTIVE CORRECTION OF A LOCAL SIGNAL ESTIMATOR SO AS TO MINIMIZE TO THE SMALLEST STATISTICAL UNCERTAINTY THE DIFFERENCE BETWEEN A PREDETERMINED SIGNAL ATTRIBUTE AND THE SAME ATTRIBUTE OF THE LOCAL SIGNAL ESTIMATOR. BECAUSE OF THE STATISTICAL UNCERTAINTY OF SIGNAL AMPLITUDE OR PHASE IN A REAL WORLD SIGNAL MEDIUM SUFFERING FROM MULTIPATH TRANSMISSIONS, THE SIGNAL ATTRIBUTE CHOSEN FOR TRACKING IS FREQUENCY. FREQUENCY IS DEFINED AS A TIME RATE OF CHANGE OF PHASE. IF IN A PERIOD OF T SECONDS A SIGNAL UNDERGOES A PHASE SHIFT OF $\Delta\theta$ RELATIVE TO OUR LOCAL REFERENCE, THEN THE FREQUENCY OFFSET IS,

$$\Delta f = \frac{\Delta\theta}{T} \quad (28)$$

IT IS OBVIOUS THAT THE SIMPLEST TRACKING STRATEGY IS TO OBTAIN THE DIFFERENCE PHASE FROM THE TWO MOST RECENT MATCHED FILTER CALCULATIONS THEN INCREMENT THE LOCAL REFERENCE FREQUENCY IN ACCORDANCE WITH RELATION (28) IMMEDIATELY PRIOR TO THE NEXT MATCHED FILTER INTEGRATION. ONE SIMPLE MEANS OF ACCOMPLISHING THIS IS SHOWN IN FIGURE 18.

THE VOLTAGE CONTROLLED OSCILLATOR (VCO) WHICH GENERATES THE LOCAL ESTIMATOR IS INCREMENTED BY THE SIGNAL IN THE ZERO ORDER HOLD (ZOH) CIRCUIT WHENEVER A CALCULATION HAS JUST TERMINATED. THE SINE AND COSINE OF THE MOST RECENT ANGLE CALCULATION IS CROSS-MULTIPLIED WITH THE STORED VALUES OF THE PREVIOUS ANGLES UNDER THE RELATION,

$$\begin{aligned} \sin \Delta\theta &= \sin(\theta_N - \theta_{N-1}) \\ &= \sin\theta_N \cdot \cos\theta_{N-1} - \cos\theta_N \cdot \sin\theta_{N-1} \end{aligned} \quad (29)$$

RATHER THAN MAKE A FULL CORRECTION OF THE MAGNITUDE OF EQUATION (28)

IT IS WISE TO REMEMBER THAT THE SIGNAL, IF PRESENT, MAY BE HIGHLY CORRUPTED WITH NOISE. THEREFORE IT IS ADVISABLE TO PROVIDE A VCO UPDATE WHICH IS A FRACTION OF THAT OF EQUATION (28). IT HAS BEEN FOUND EXPERIMENTALLY THAT A GOOD VCO UPDATE VOLTAGE PROFILE IS AS SHOWN IN FIGURE 19. A LINEAR RELATIONSHIP BETWEEN Δf UPDATE VOLTAGE AND $\Delta\theta$ IN ACCORDANCE WITH EQUATION (28) IS MAINTAINED FOR A RANGE ABOUT ZERO. BEYOND THAT THE Δf UPDATE IS MAINTAINED CONSTANT AT A VALUE,

$$\Delta f = \left(\frac{\Delta\theta_m}{T} \right) (\text{SGN} \Delta\theta) \quad (30)$$

WHERE $\text{SGN}(x)$ IS THE SIGNUM FUNCTION.

FORTUNATELY SUCH A RELATION AS (30) MAY BE READILY GENERATED BY PASSING THE $\text{SIN} \Delta\theta$ VOLTAGE THROUGH A LINEAR CIRCUIT WHICH LIMITS IN THE MANNER SHOWN BY THE DASHED LINES IN FIGURE 19.

MORE ELABORATE TRACKING STRATEGIES MAY BE UTILIZED SUCH AS MAINTAINING A RUNNING RECORD OF SUCCESSIVE $\Delta\theta'_i$ AND UPDATING THE VCO EACH TIME BY A WEIGHTED SUM,

$$\Delta f_{\text{UPDATE}} = \sum_i m_i \Delta\theta_i \quad (31)$$

OR, AS CITED UNDER DETECTION, A FREQUENCY TRAJECTORY MAY BE INFERRED FROM A RUNNING MEMORY OF SUCCESSIVE VECTORS AND AN UPDATE STRATEGY CHOSEN SO AS TO BRING THE OBSERVED FREQUENCY TRAJECTORY INTO ALIGNMENT WITH A PARTICULAR TRAJECTORY FROM A GIVEN DICTIONARY. AS AN EXAMPLE OF THIS, FIGURE 20 IS THE FREQUENCY-TIME TRAJECTORY OF A FIXED FREQUENCY SOURCE MOUNTED ON A VEHICLE WITH UNIFORM VELOCITY IN A *FLYBY* MODE. THE FREQUENCY OF THIS SOURCE IS f_0 . AT LARGE NEGATIVE TIME THE RECEIVER PERCEIVES A FREQUENCY $f_0 + \delta f$ DUE TO APPROACHING DOPPLER WITH CONVERSE RELATION AS THE VEHICLE RECEDES. IN THE EVENT THAT AN INTERCEPT

DISCLOSED A SET OF FREQUENCIES AT THE TIMES t_1, t_2, t_3, t_4 WHICH SHOWED A HIGH LIKELIHOOD OF BELONGING TO THIS TRAJECTORY, THEN THE BEST STRATEGY MIGHT BE TO WEIGHT THE OFFSET TO THE LOCAL FREQUENCY ESTIMATOR SO AS TO BE OF VALUE Δf AS SHOWN IN FIGURE 20. THIS PROCESS WOULD THEN CONTINUE SO LONG AS THE CONDITIONS OF TRACKING THIS TRAJECTORY WERE MET.

MANY VARIATIONS ON THESE METHODS ARE POSSIBLE. HOWEVER, THE STATISTICAL LIKELIHOOD OF A SIGNAL PHASE FLIP DUE TO THE PROPAGATION MEDIUM MUST BE ASSESSED FOR ANY LONG STATISTICAL SAMPLES.

ONE POWERFUL ADVANTAGE TO THE USE OF THE SIMPLE $\Delta\theta$ VALUES FOR FREQUENCY UPDATING IS THAT NO PENALTY IS ACCRUED SHOULD A SIGNAL DISAPPEAR. UNLIKE PHASE OR AMPLITUDE TRACKING MEANS, THERE IS NO TENDENCY TO INCUR UNCONTROLLED *OUT OF LOCK* BEHAVIOR WITH NOISE ONLY. THIS IS BECAUSE THE TRACKING METHOD IS SUCH AS TO MINIMIZE THE MEAN OF THE DIFFERENCE ANGLE STATISTICS. IF ONE IS JUDICIOUS IN HIS CHOICE OF VCO UPDATE, A NOISE ONLY SITUATION WITH ITS ZERO MEAN IS IDENTICAL IN EFFECT TO ON-FREQUENCY TRACKING. IF ONE HAD BEEN TRACKING A SIGNAL WHICH DISAPPEARED THE FREQUENCY TRACKER WILL *MARK TIME* AS IT WERE WITH A RANDOM FLUCTUATION ABOUT THE PREVIOUS SIGNAL FREQUENCY. WHEN THE SIGNAL AGAIN REAPPEARS THE MEAN OF THE DISTRIBUTION WILL INDICATE AN OFF-FREQUENCY CONDITION AND THE FREQUENCY TRACKER WILL BEGIN ITS STATISTICAL DRIFT TOWARD A CORRECTION. ALL OF THIS IS INDEPENDENT OF THE ABSOLUTE PHASE OF THE INCOMING SIGNAL.

ONE POINT MUST BE MADE CONCERNING THE SPECTRAL PURITY OF THE RECEIVED SIGNAL. THE METHOD DESCRIBED FOR FREQUENCY TRACKING MAKES USE OF THE EXISTENCE OF A STATISTICAL MEAN FOR INDICATION OF OFF-FREQUENCY CONDITIONS. IF THE SIGNAL IS BADLY SMEARED IN FREQUENCY IN A NOISE-LIKE

SENSE THEN THE FREQUENCY TRACKER WILL STILL FUNCTION, ALTHOUGH AT REDUCED EFFICIENCY, IF A LEGITIMATE COHERENT MEAN EXISTS. IF, ON THE OTHER HAND, AN OTHERWISE SHARP SPECTRAL LINE SPLITS INTO MULTIPLE LINES, THE TRACKER WILL FOLLOW THE MEAN FREQUENCY OF THESE LINES.

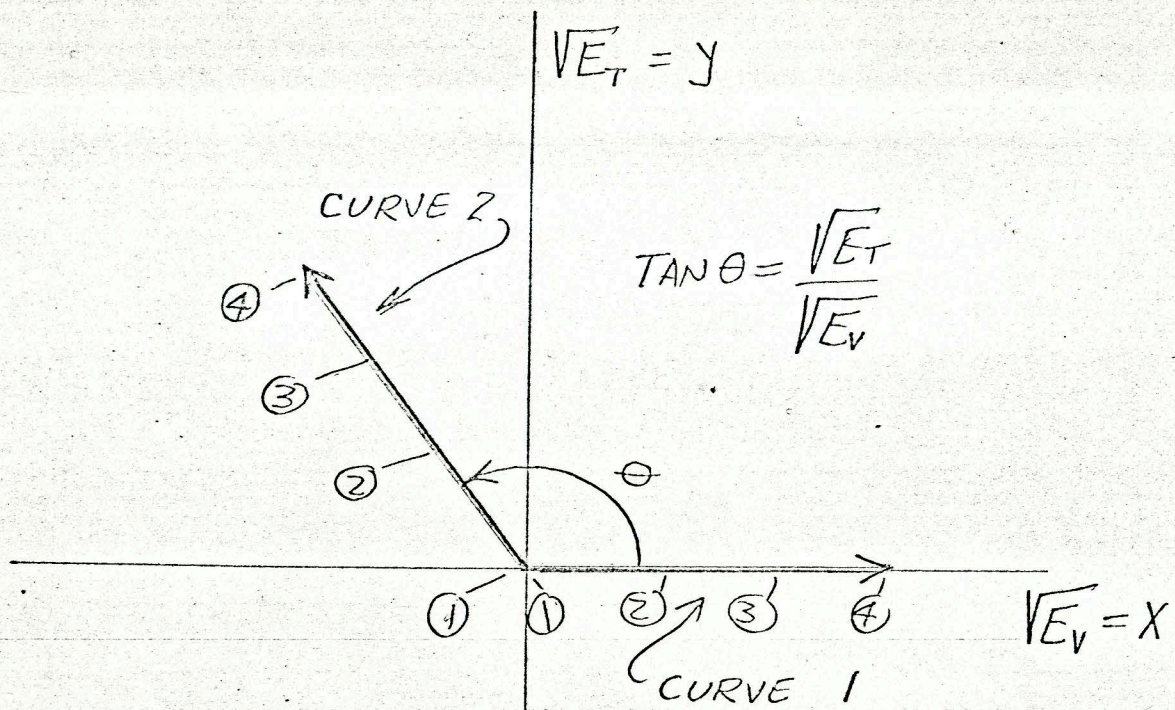


FIGURE 1

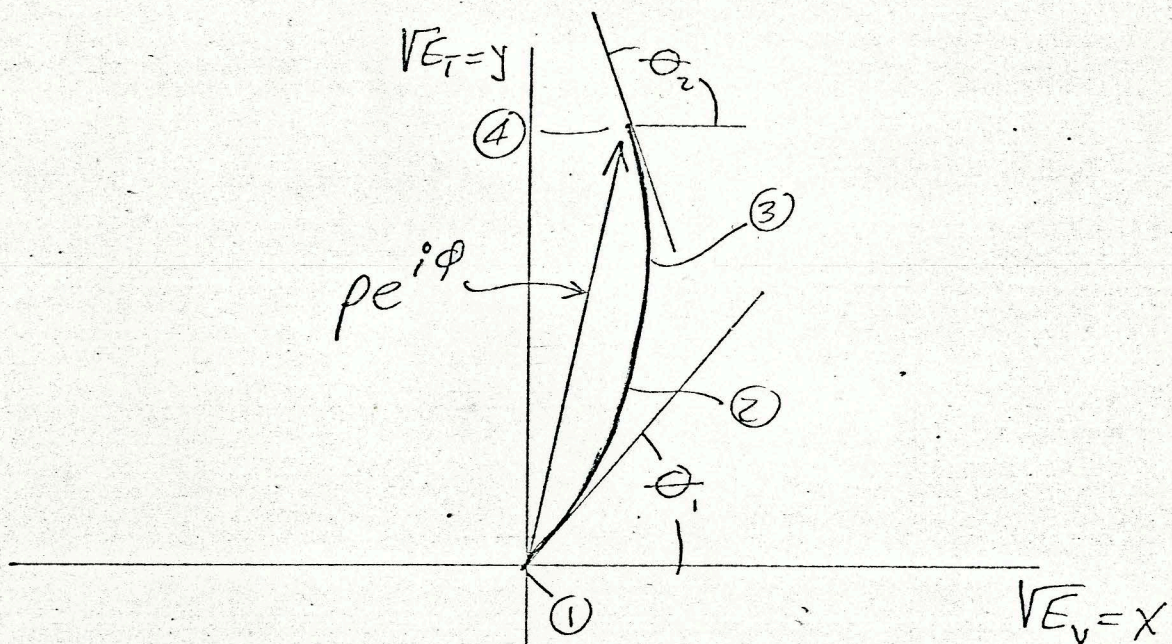


FIGURE 2

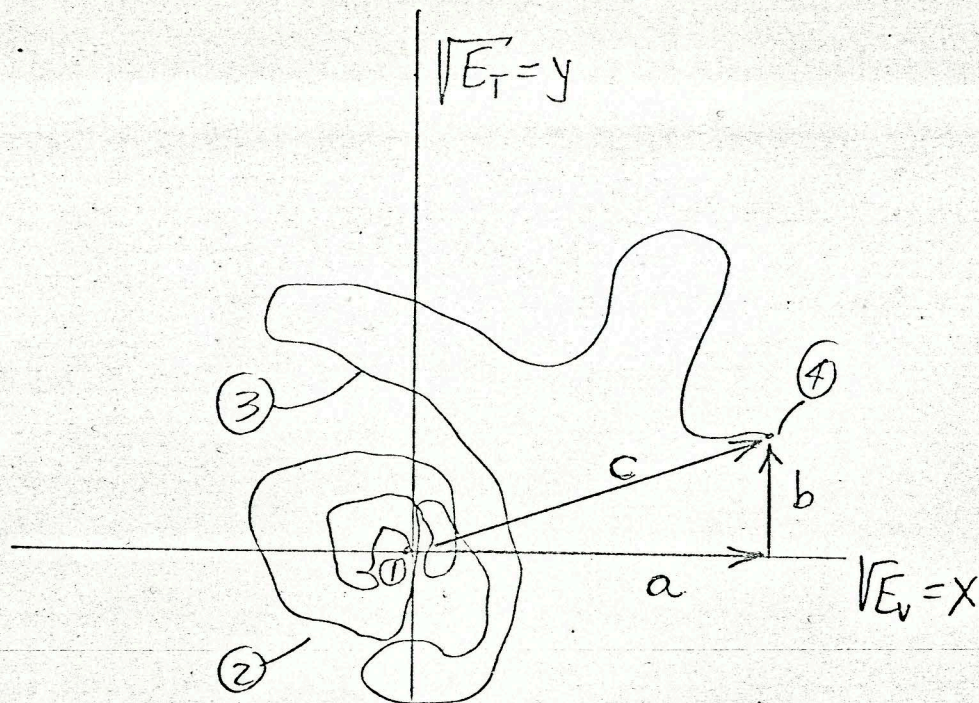


FIGURE 3

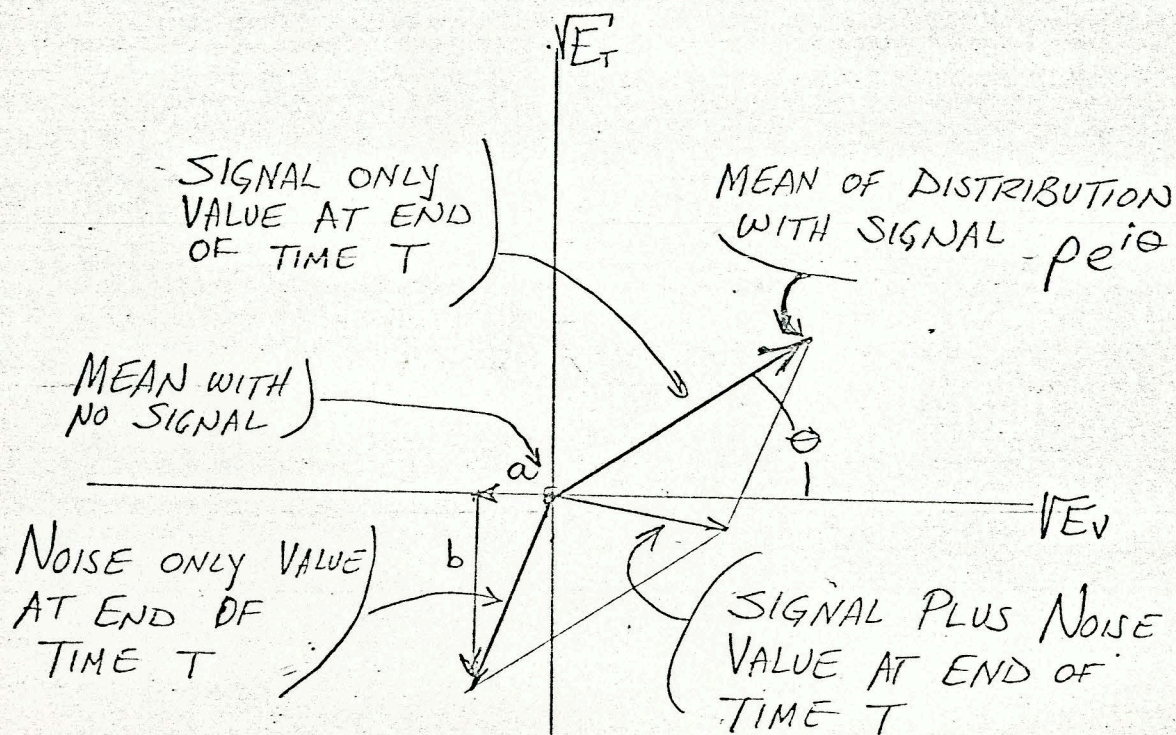
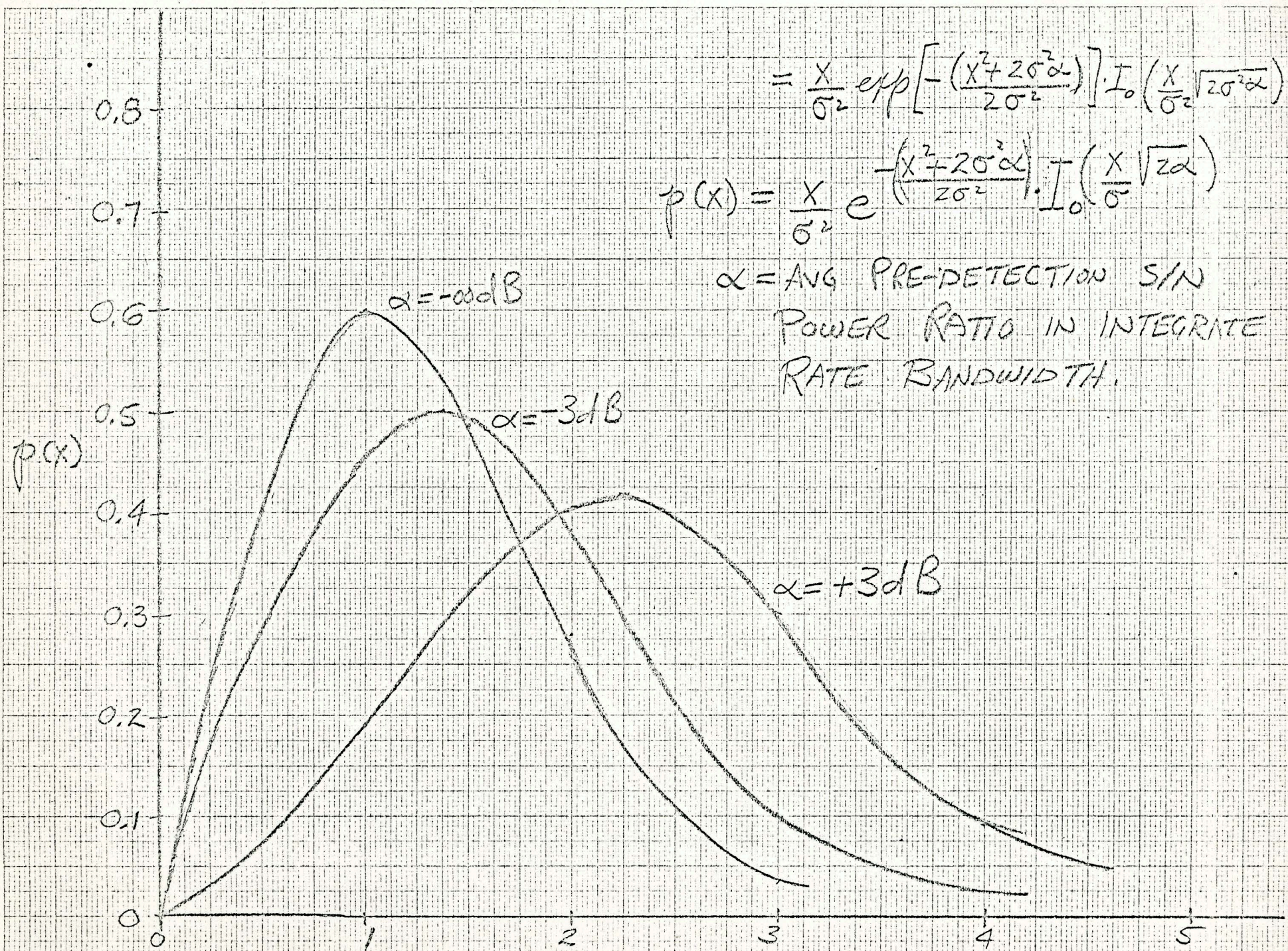


FIGURE 4



$$= \frac{x}{\sigma^2} \exp\left[-\frac{(x^2 + 2\sigma^2\alpha)}{2\sigma^2}\right] \cdot I_0\left(\frac{x}{\sigma^2} \sqrt{2\sigma^2\alpha}\right)$$

$$p(x) = \frac{x}{\sigma^2} e^{-\frac{(x^2 + 2\sigma^2\alpha)}{2\sigma^2}} \cdot I_0\left(\frac{x}{\sigma} \sqrt{2\alpha}\right)$$

α = AVG PRE-DETECTION S/N
POWER RATIO IN INTEGRATE
RATE BANDWIDTH.

x = AMPLITUDE IN UNITS OF σ

FIG 5

$$p(\theta) = \frac{e^{-\alpha}}{2\pi} + \frac{1}{2} \sqrt{\frac{\alpha}{\pi}} \cos \theta e^{-\alpha \sin^2 \theta} [1 + \operatorname{erf}(\sqrt{\alpha} \cos \theta)]$$

α = AVG PRE-DETECTION S/N POWER RATIO IN INTEGRATE RATE BANDWIDTH

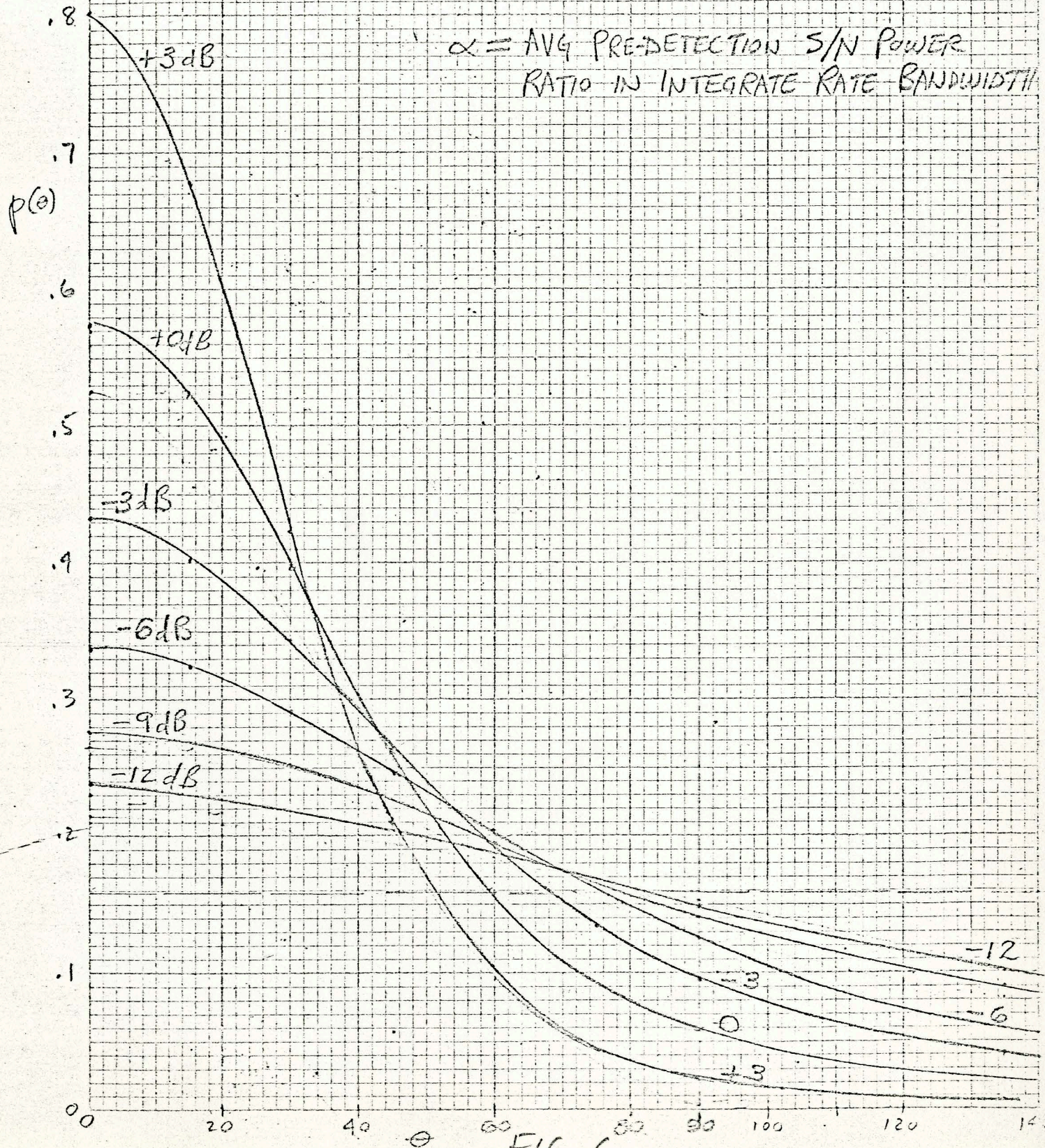


FIG 6

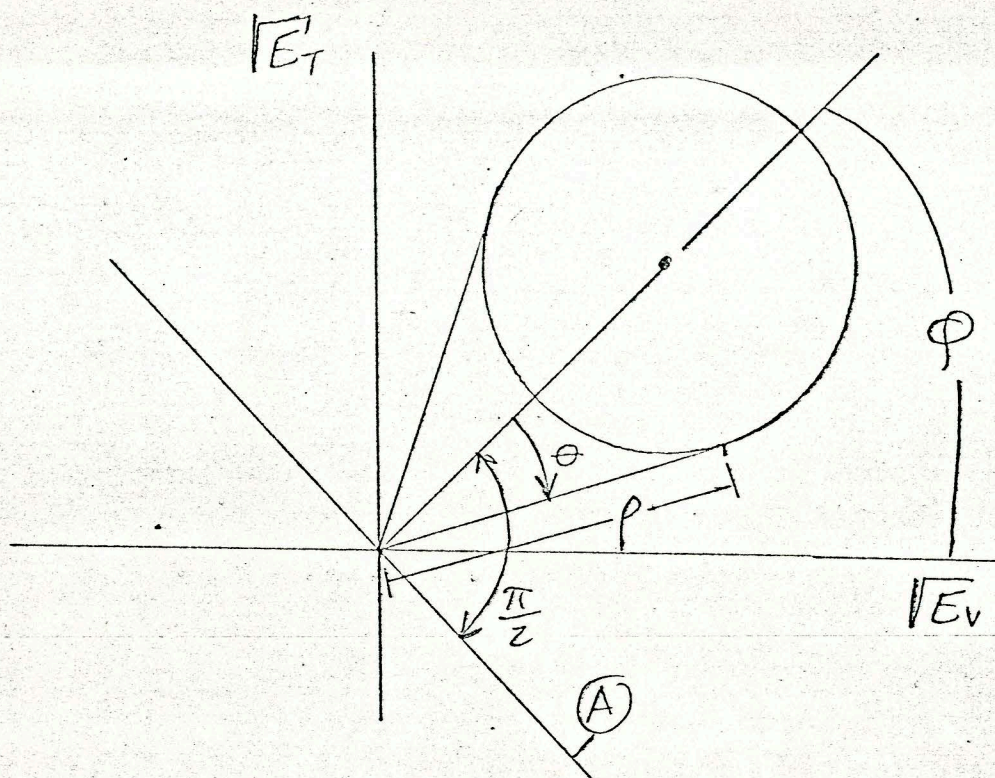


FIGURE 7

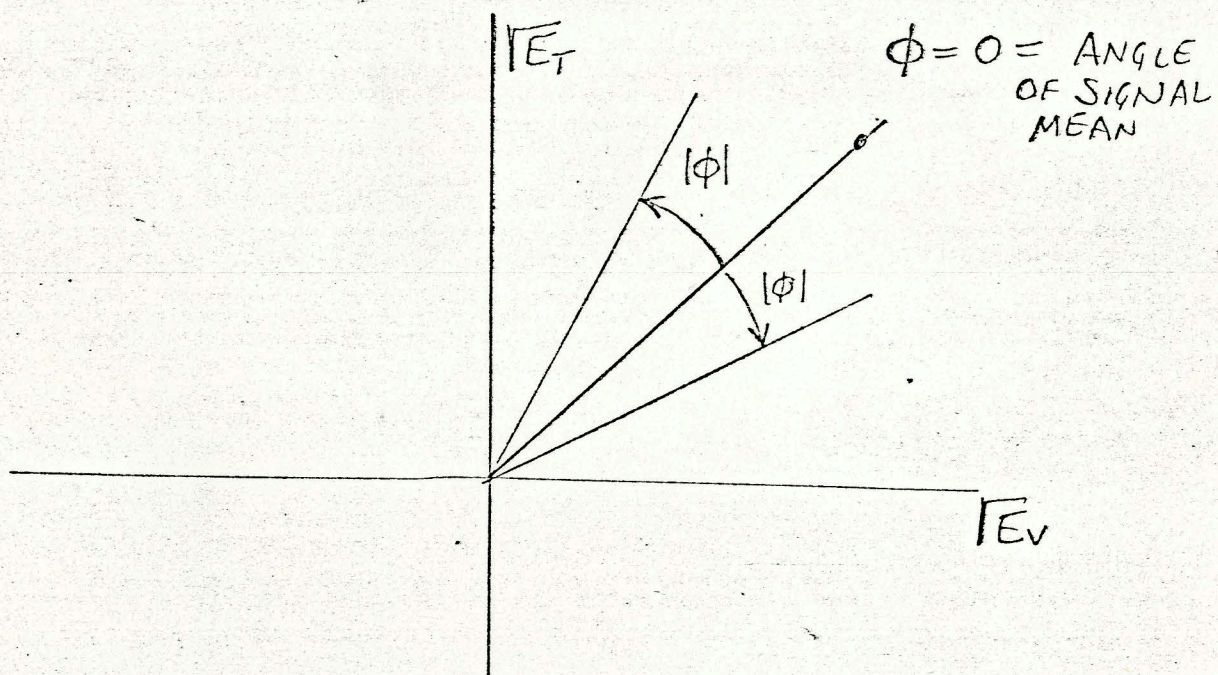


FIGURE 8

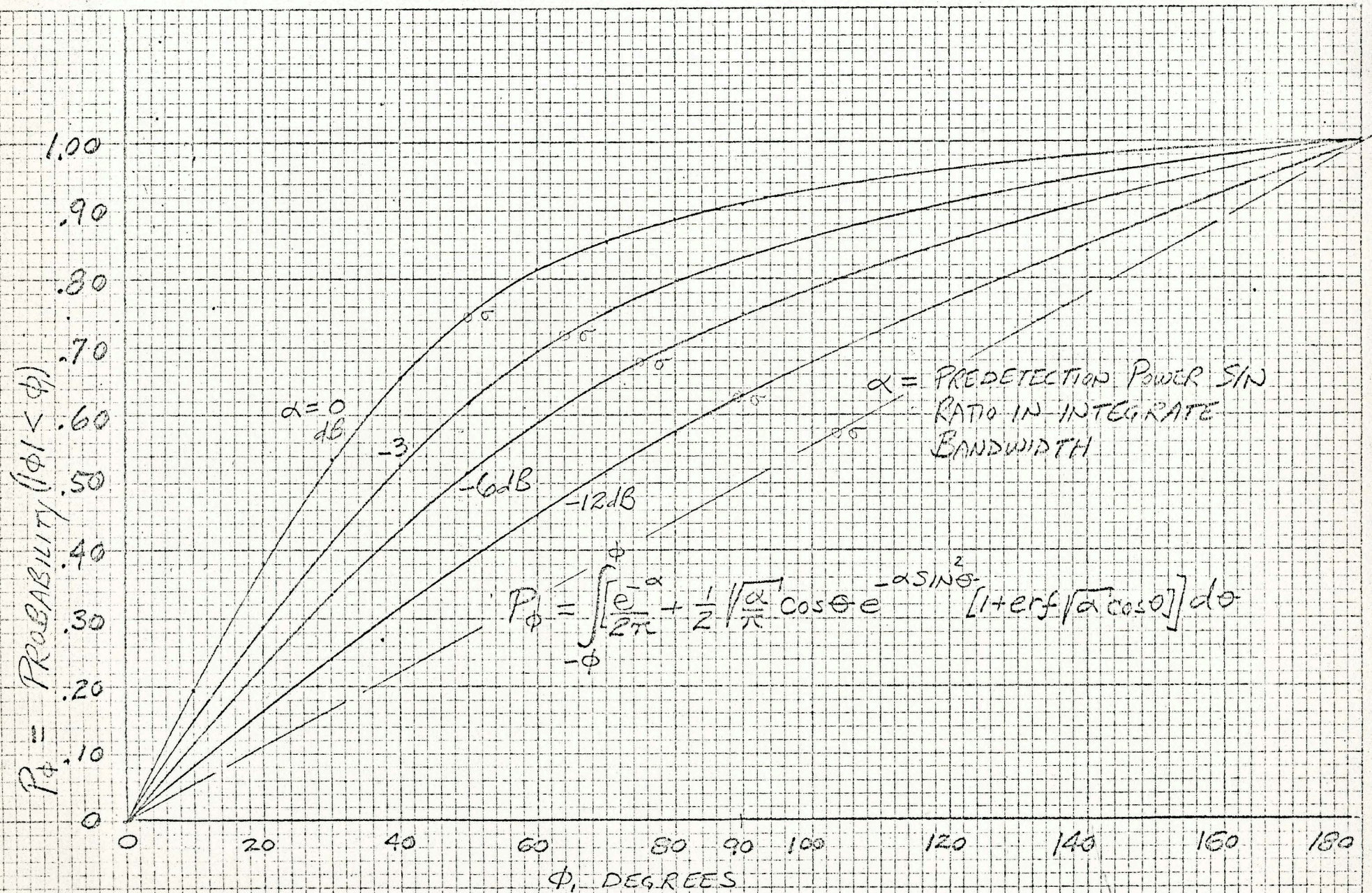


FIG 9

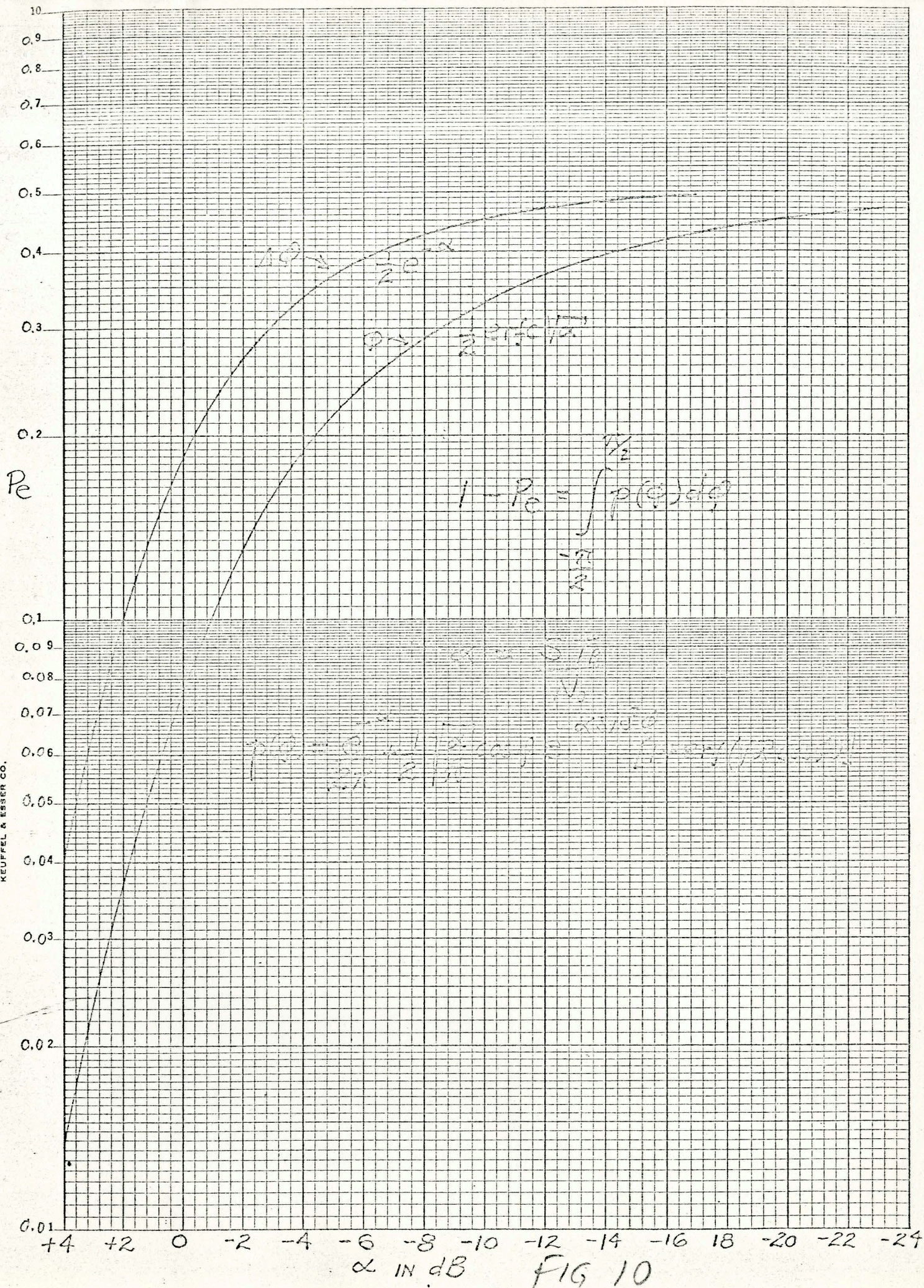


FIG 10

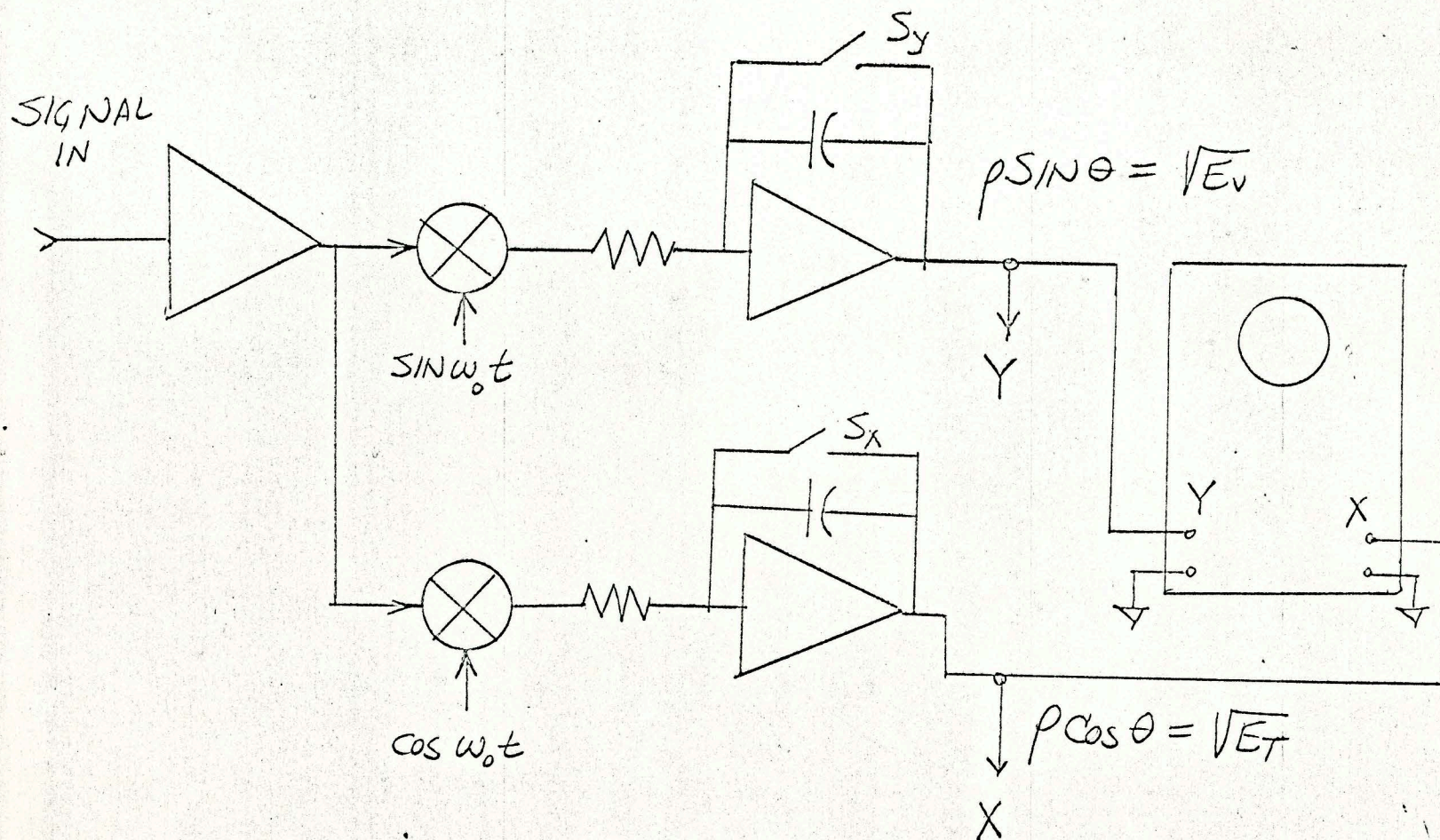


FIGURE 11

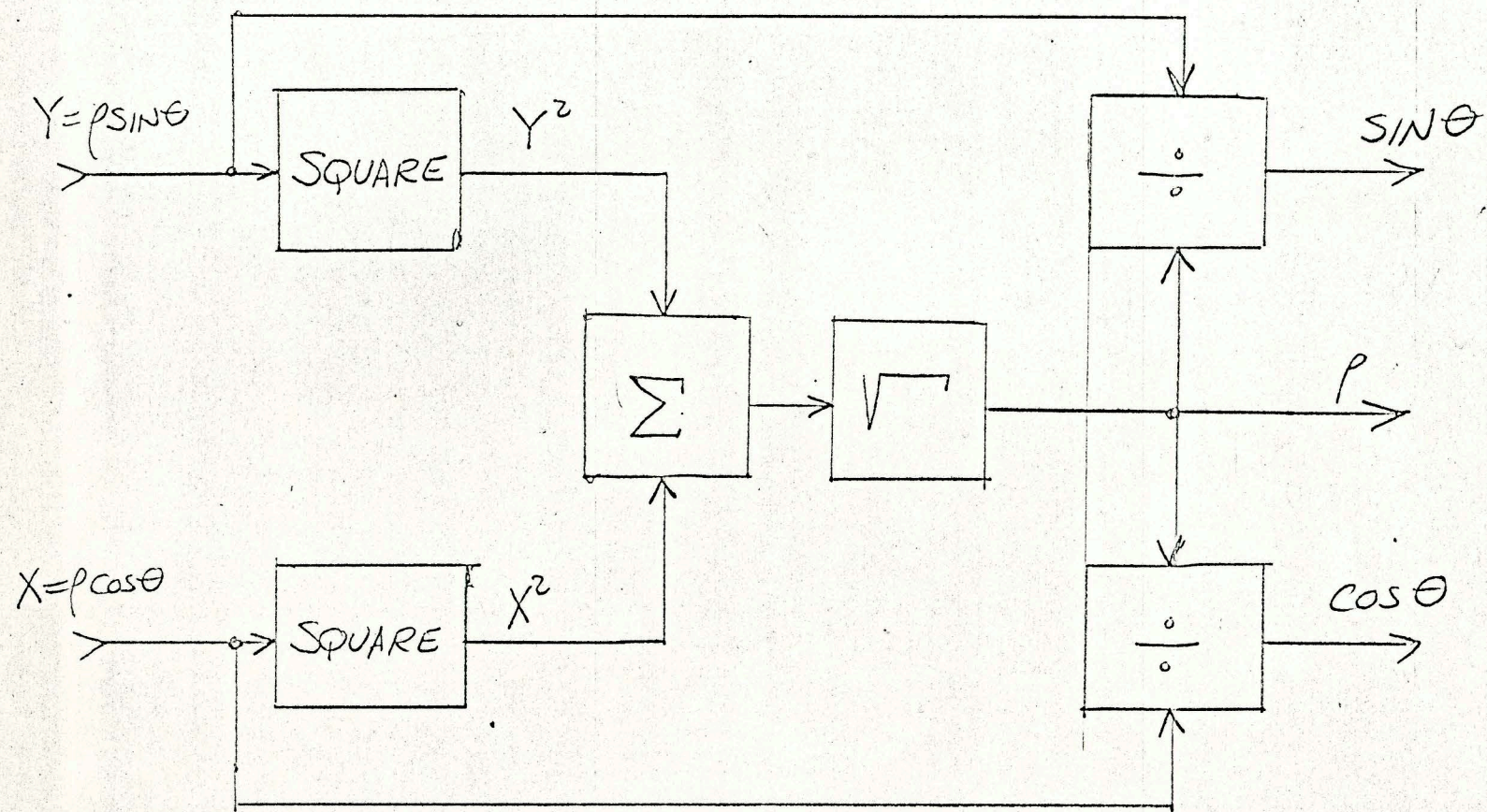


FIGURE 12

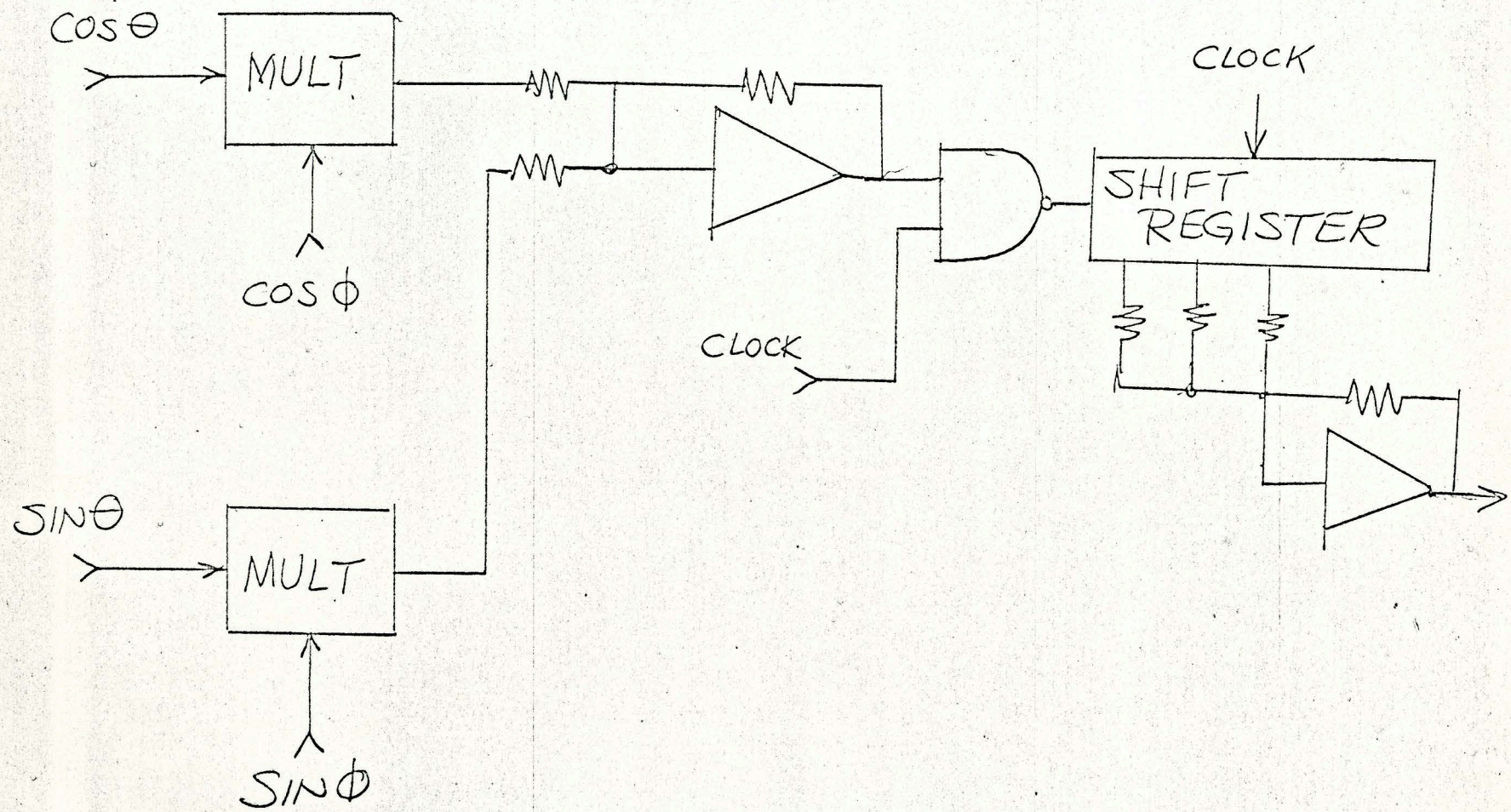


FIGURE 13

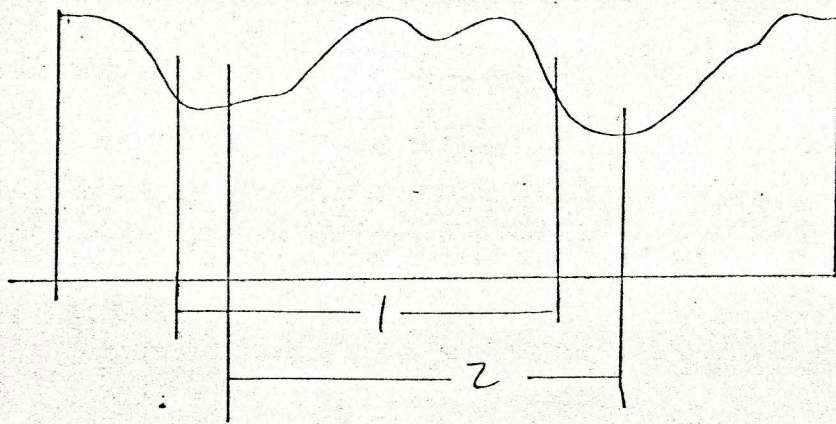


FIGURE 14

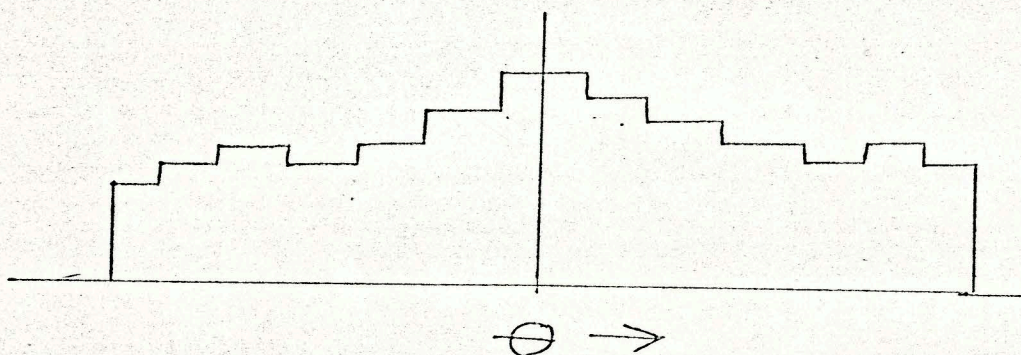


FIGURE 15

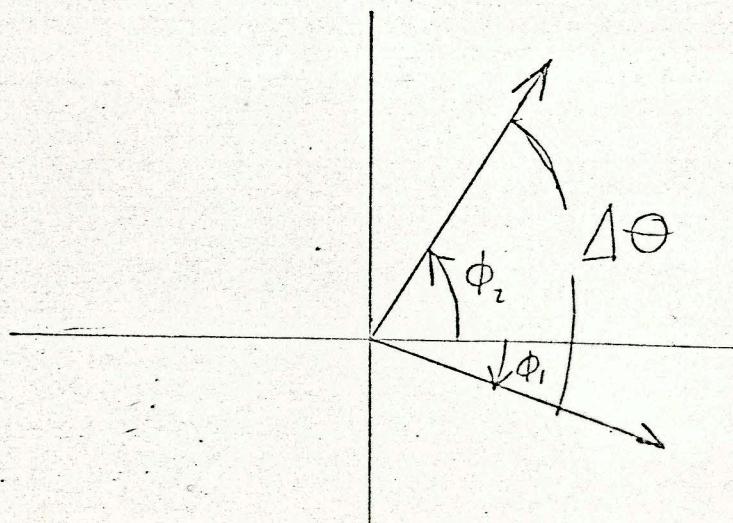
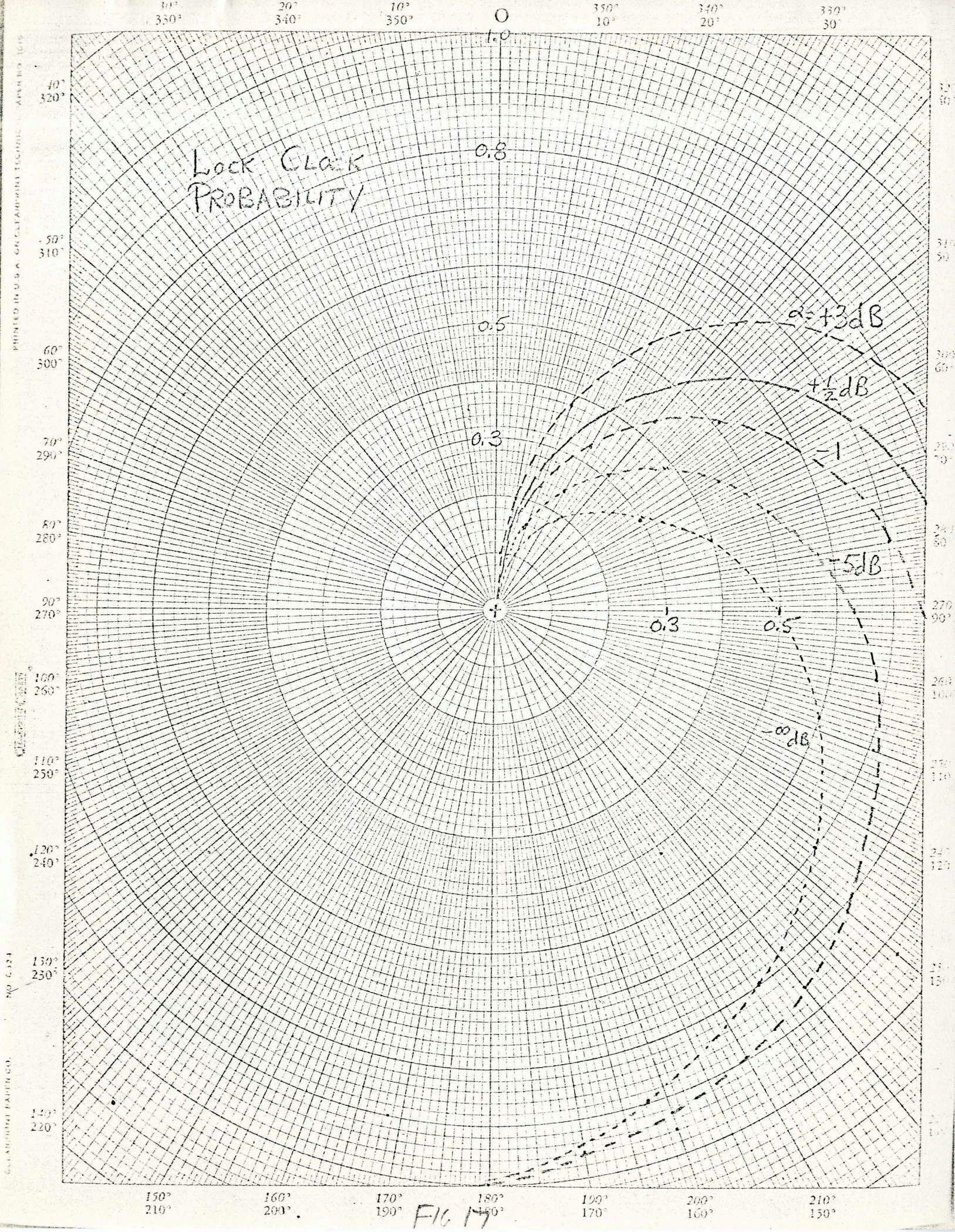


FIGURE 16



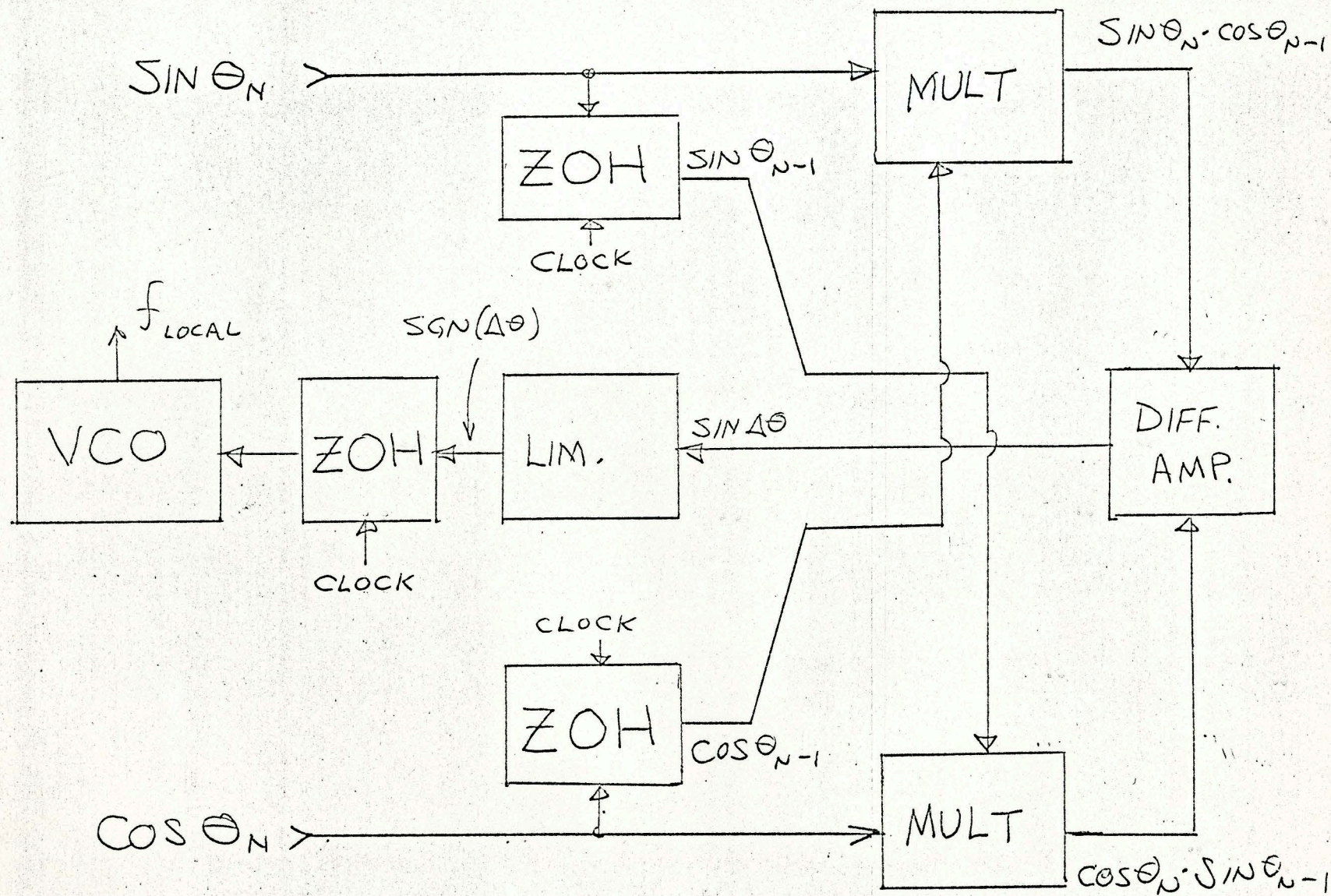


FIG 18

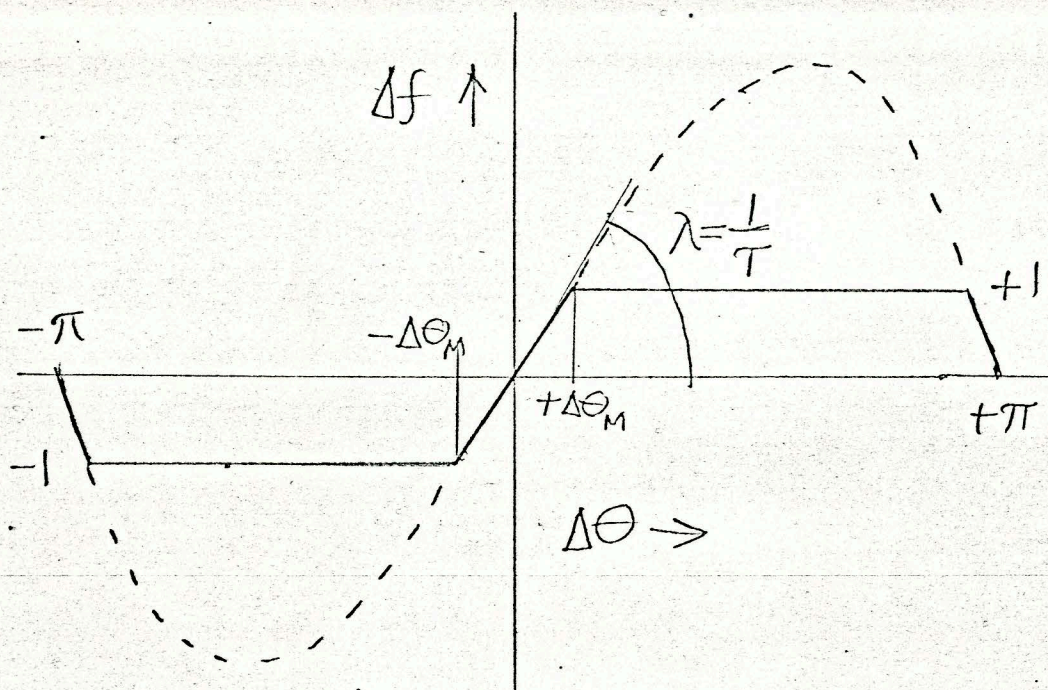


FIGURE 19

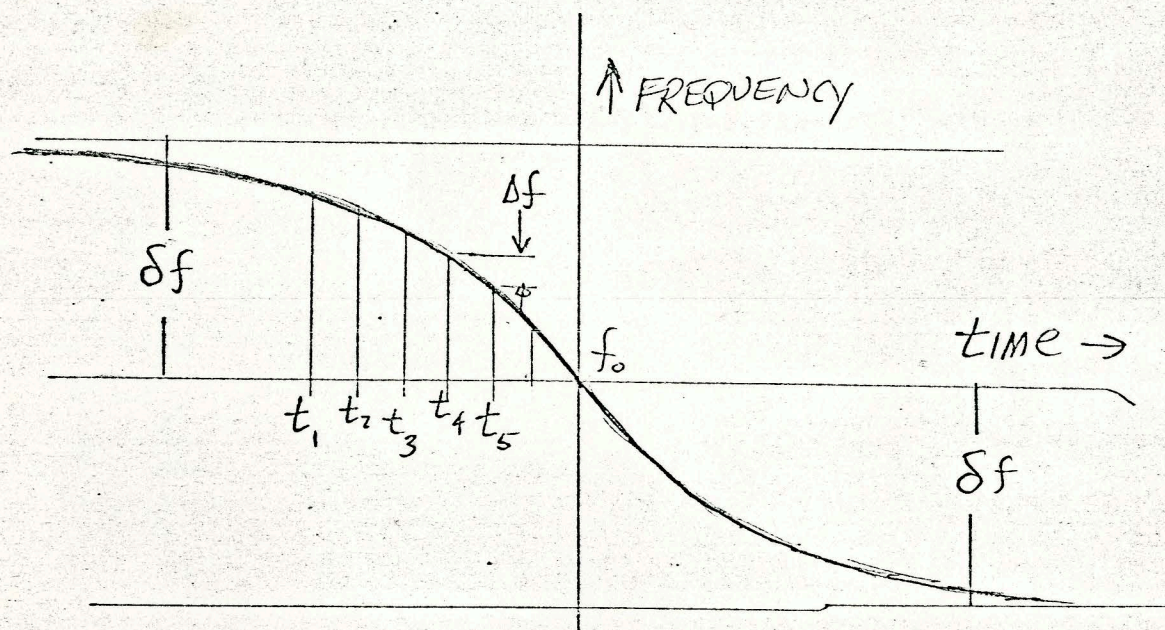


FIGURE 20