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Reducing Uncertainty

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REDUCING UNCERTAINTY

The purpose of this communication is to bring to the attention of audio engineers an important fact about a commonly misapplied theoretical concept. This concept is called the uncertainty principle. There is nothing wrong with the concept; what is wrong is that it is so simple in form that it is freely applied TO ENGINEERING PROBLEMS without regard to the errors which may result due to lack of understanding of its derivation. ~~or how to apply it~~.

If one enters into an observation of a signal with absolutely no prior information concerning the state of the signal, he is limited by the uncertainty principle in the fineness of detail to which he may describe the joint time and frequency properties of that signal. Prior information which one may possess about the signal reduces the number of possible states which may exist at the moment of observation. Many of the signals used by audio engineers fall into this latter catagory. If one has a properly limited number of states at the moment of observation, the bounding of detail is not governed by the principle of uncertainty. The significance of this statement demands a definition of uncertainty, a term very commonly misapplied in this context.

Under appropriate conditions, when two dimensionally reciprocal parameters, such as time and frequency, represent domains of description of a signal, the functions described by these parameters bear a Fourier transform relationship. [1], p235, ~~[2]~~ A loudspeaker's impulse response and its frequency response are such functions. [3] The signals which bear a Fourier relationship are generally complex functions of a real variable such as time or frequency. A measure of the distribution of the signals with respect to the variable is afforded by evaluating the relationships called, by analogy with static analysis, the moments of the distribution. The zeroeth moment crudely relates to the net amount of signal, the first moment relates to the mean or expectation value, and the ~~third~~ SECOND moment relates to the variance or "spreading" of the signal. In deciding what signal attribute is to be evaluated for its moments it is common to use the formalism of Quantum mechanics. This uses the moments of the square of the absolute value of the defining signal, thus equating to what is considered the "power" contained in the signal. A footnote of historical significance is that Gabor was forced to introduce a complex signal (now called the analytic signal) in order to bring this formalism of quantum mechanics to signal theory in deriving the uncertainty principle. [4] p433

The positive square root of the mean square deviation from the expectation value, normalized for total energy, is called the uncertainty of the signal. Thus uncertainty is the r.m.s. spread of the signal from its mean and is a fixed number. A straightforward mathematical manipulation then produces the signal theory equivalent to Heisenberg's uncertainty relation: the product of the uncertainty in time and the uncertainty in frequency is greater than or equal to a constant. This constant is one half, if one uses the dimensions of seconds and hertz. FOR TIME AND RADIANS PER SECOND FOR FREQUENCY

The reason for the inequality is that the variance based on signal power is not a unique property. Many different signals may have the same uncertainty. This is because signal phase, although assumed to exist for calculation purposes, does not uniquely enter into calculation of the variance.

When properly applied, the uncertainty relationship is of great value and establishes the bandwidth required to define, in a mean square sense, a signal of restricted duration. It is unfortunately much too easy to misapply this principle if the definition of uncertainty is not observed. Now consider the restrictions imposed in the derivation of this principle. The moments with respect to the power function were assumed to be defined only for time or frequency, not jointly. If one is thus forbidden to consider the frequency dependence of a time function the principle must stand unchallenged as representing a true limit. In the overall grand picture of things this is what one must generally do. However, engineers quite frequently work in a more restricted fashion and are, among other things, only interested in those effects yielding results within the observational limits of equipment or sometimes within what is called engineering accuracy. Accordingly, the engineer may be privileged to possess relatively more information concerning a process than allowed by the restrictions imposed in defining the uncertainty relation. If, in particular, one can specify uniquely the joint time-frequency dependence of a signal he can avoid the restrictions of the uncertainty principle.

It has been shown in a previous publication in this journal [3] that there is a special class of signal description for which there is a unique and single valued time delay as a function of frequency. Signals possessing this property are complex with constant amplitude when evaluated as a function of frequency. When describing a transmission system such signals are said to be all-pass. It was further shown that more complicated transmission systems, of the type generally of interest to engineers, could be expressed as a superimposed sum of simple all-pass systems. This means that it is possible to transform from a frequency-only coordinate system to a special joint time and frequency coordinate system in which the original signal is now composed of a sum of simple curves of known shape. When one has the set of curves corresponding to this linear sum he may write down, by inspection, either the time domain or frequency domain equivalent without the necessity to use an integral transformation. An integral transform would still give the correct answer but the point is that the properties of many of the systems analyzed by engineers allow for the more economical joint time-frequency expansion. Even when the systems are exceedingly complicated, as for example loudspeakers, an engineer may still be able to handle the situation by concerning himself with those terms of the greatest engineering significance.

Considering the uncertainty principle it should be apparent that if one can specify the joint time-frequency behavior to ever increasing accuracy, he might not be terribly concerned about the product of second moments. As a crude example of this, suppose one is asked to specify the length of time required to determine the angular frequency of the minute hand of his watch bounded to a value between stopped and running twice normal rate. One of course has prior knowledge that the hand is equivalent to a vector with constant amplitude and frequency to be determined. This is the nature of prior information for the equivalent of a joint time-frequency coordinate. The basid frequency is of the order of 300 microhertz. Certainly no one would assert that it takes 15 minutes of observation to determine that his minute hand was bounded between a stopped position and running twice normal rate. Yet this is the result if we blindly used the uncertainty relation. A little thought will show that if we know the angle as a continuous function of time, and all the derivatives exist, we can know the angular rate as accurately as we choose in a time interval as short as we choose. Taking this analogy into the form more suited to audio engineering, it takes a vanishingly small time to determine the frequency of a signal which is known to be of constant amplitude for the period required for observation. Successive phase values of the analytic signal is all that is required. [2] p739

This is not an anomaly or a refutation of the uncertainty principle, but an observation that if we possess more information than assumed in the original derivation, we may use that information to reduce the lack of knowledge of our experiments below that presumed allowed by the uncertainty principle. It is my hope that bringing this common sense fact to the attention of audio engineers will allow some to avoid an all-too-common pitfall.