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Energy Loss Through Storage

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Sir,

I respectfully submit the enclosed manuscript for consideration as a Report in *Science*. Two manuscript copies are included, along with two copies of the more important cited references to assist the review process.

Sincerely,

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Abstract. *The process of storing energy under equilibrium conditions loses work that could have been obtained directly from the source of energy. This is independent of the form of the energy and can be extended to the analysis of situations otherwise difficult to formulate in a common description, such as multiple energy conversions involving solar, electrical, chemical, and mechanical storage.*

Conservation of total energy is a basic principle of physics for which no counterexample has ever been observed. The present analytic treatment of energy related phenomena uses terminology derived from what are called intensive and extensive parameters *(1)*. Such parmeters do not themselves derive from the conservation of energy but are assembled in any particular energy related description in such a way as to be consistent with this conservation *(2)*. This terminology, while useful in describing energy related phenomena, makes it difficult to identify general conditions which the principle of conservation of energy may impose on our observations of such phenomena. The theory presented in this correspondence starts from the principle of conservation of energy and develops the concept of a generalized frame of reference from this conservation; then conditions are developed for descriptions made in such a frame of reference. It will be shown that this leads to the result that it takes energy to store energy. Specifically it will be shown that if a linear storage reservoir is connected to a source of energy and allowed to come to equilibrium with that source, then the total amount of stored energy capable of performing work is half the amount extracted by the reservoir from the source.

If there is a finite scalar entity E, identified as total energy, which is to remain invariant when all aspects of a process are considered over some frame of reference s, then we can postulate the existence of a total energy density E(s) such that the net sum of this density over all values of this frame of reference is equal to the total energy. This can be expressed as,

$$\int_s E(s)\, ds = E. \qquad (1)$$

The set of all frames of reference s for which this invariance holds will be defined as generalized coordinates of energy measure. When energy representations in generalized coordinates s_i are mapped to energy representations in generalized coordinates s_j under conditions of conservation of total energy E, then

$$\int_{s_i} E(s_i)\, ds_i = \int_{s_j} E(s_j)\, ds_j = E. \qquad (2)$$

As it stands, this expression is of little utility, but if we further define a new complex entity h(s), the square of whose magnitude is equal to E(s), then we can cast relation (1) into a form that is known to be of use in descriptions of energy related events. Defining

$$h(s) = f(s) + i\, g(s), \qquad (3)$$

we have,

$$\int_s |h(s)|^2 ds = E. \qquad (4)$$

Since E is finite, h(s) is square integrable, and consequently will be of class L^2 if ds is the Lebesgue measure of the frame of reference s *(3)*.

The additional geometric structure imposed by relation (4) involves a linear scaling relationship between total energy content and the magnitude squared of h(s). While this restricts our considerations to those frames of reference in which this condition prevails, it does not significantly reduce the usefulness of the approach since the application of descriptions of Lebesgue square integrability is well known both in classic physics *(4)* and in quantum physics *(5)*. Thus we may make use of the extensive mathematical literature concerning linear spaces of class L^2. Of particular benefit is a theorem due to Titchmarsh which identifies the necessary and sufficient condition under which a complex h(s) will be analytic and of class L^2 *(6)*. The condition is that f(s) and g(s) be conjugate functions related to each other through the Hilbert transform.

The Hilbert transform is a mapping relationship that appears in several forms in our existing analysis. It can be shown to derive from Cauchy's integral formula *(7)* and appears as the so-called dispersion relations of systems expressed in coordinates of angular frequency *(8)*. This brings out the interesting result that causality is imposed on any linear frame of reference identified with the constancy of total energy.

The Hilbert transform is of strong L^2 type *(9) (10)*, which means that not only is

$$|h(s)|^2 = [f(s)]^2 + [g(s)]^2, \qquad (5)$$

but that

$$\int_s |f(s)|^2 ds = \int_s |g(s)|^2 ds \qquad (6)$$

everywhere except over sets of Lebesgue measure zero. The result of this is that when we impose L^2 structure on our analysis the expression which is identified as total energy density is partitioned into two terms,

$$E(s) = V(s) + T(s),$$

where

$$E(s) = |h(s)|^2$$

$$V(s) = [f(s)]^2$$

$$T(s) = [g(s)]^2. \qquad (7)$$

A further result is that if we integrate over the whole of space s in order to evaluate the scalar partitions of E which is imposed by this additional structuring, then,

$$E = V + T$$

and

$$V = T = \tfrac{1}{2}E, \qquad (8)$$

where

$$V = \int_s V(s)ds$$

$$T = \int_s T(s)ds.$$

Relation (7) indicates that a linear complex representation is sufficient to identify energy density and its partitioning into two components, and relation (8) identifies the net available work of each component as half the total energy. While the frame of reference s derives from the constancy of total energy, we can recognize the significance of V(s) and T(s) by identifying their counterpart in simple mechanical or electrical situations. An earlier paper which derived these equations identified V(s) and T(s) as potential energy density and kinetic energy density respectively *(11)*. The terminology must be taken in context with the frame of reference s, since V(s) and T(s) are both expressed in the same set of coordinates. If, for example, s is a frame of reference identified uniquely with potential energy, then T(s) is the way kinetic energy density is manifest in this potential energy frame of reference. The earlier paper went into a considerable discussion of relation (7) and will not be further elaborated in this correspondence, but the extended value of relation (8) was not fully appreciated when the paper was first published. New considerations concerning relation (8) and its significance to energy storage are presented here.

Relation (8) indicates that the storage of energy, if allowed to go to completion in a frame of reference identified with potential energy, leaves a value V which is half the total energy giving rise to V. The other half appears as a term identified as T. Work T is required to establish the storage configuration V. A literal interpretation of this relationship is that it takes energy to store energy. In order to divert energy to a stored form it is necessary to have an agent of energy transport, and this agent exacts an energy toll.

Equal partitioning into the scalar terms V and T depends upon integration over the whole of s. Integration of V(s) and T(s) over part of s need not result in equal partitioning. This is particularly important when considering the consequences of using an energy reservoir between an energy source and an energy sink. If an energy reservoir is cycled between operations of depletion into a sink and equilibrium restoration from a source, then the depth of depletion determines the net efficiency of energy storage between source and sink. Total depletion of the reservoir results in a 50% conversion efficiency, according to relation (8). It is possible to raise the efficiency of transfer by partial depletion of the reservoir followed by replenishment for subsequent use, an operation known as shallow cycling, but the maximum efficiency for any finite depletion can never achieve the ideal 100% due to the residual component of T left by integrating over a finite interval.

Relation (4) is based on linear considerations and does not hold in nonlinear systems. However, many nonlinear systems can be represented by the combination of some equivalent linear component of energy density plus a nonnegative component that accounts for the net behavior. Although the efficiency of energy storage for such a combined nonlinear system may either be much higher or lower than that of a purely linear system, the existence of the linear component will limit the maximum achievable storage efficiency to a value less than 100%.

An example of the equal partitioning relationship in a linear system can be found in the case of charging a capacitor. The maximum efficiency with which a linear capacitor may be charged from a perfect battery is 50%. If an initially uncharged capacitor C is connected to a potential v and allowed to come to equilibrium, the stored energy in the capacitor is $\frac{1}{2}Cv^2$ while the work done by the battery is Cv^2. This condition prevails even if a resistor is used to restrict the maximum surge of current. One always finds that $\frac{1}{2}Cv^2$ is dissipated in the resistor, even if its value is allowed to go to zero.

The traditional explanation of this apparent paradox puts the missing half of the energy in the radiation resistance offered by the connecting wires, although the reason why such perfect partitioning takes place independent of wire configuration or dissipating resistor is not suggested. Since this is an equilibrium condition, relation (8) prevails, and we should expect half the energy to be stored in the capacitor. A resistor, or any such component in series with the capacitor, provides the second part of the analytic complex expression, relation (3), necessary to define the finite invariant entity we call energy.

Another example of relation (8) can be found in the case of a parallel sided gravity reservoir whose cross section area is constant with height. If fluid is transported under constant pressure into such a storage reservoir, equilibrium will be achieved for a height of fluid whose gravity pressure equals that of the source. Since the stored pressure is proportional to the height of the fluid remaining in the reservoir, the total amount of work which can be obtained from the reservoir fluid is half that which was required to transport the fluid into the reservoir at constant pressure.

Nonlinear capacitors, whose capacitance is a function of voltage, or sloping sided gravity reservoirs, whose cross section area is a function of the height of the fluid, may have storage efficiencies other than 50%. Simple calculation shows that the highest efficiencies are obtained for nonlinear configurations that produce depletion forces which vary in the same manner as the forces exerted during storage from the initial energy source. In the case of nonlinear capacitors, for example, the highest charge-discharge efficiency for constant resistance loads will occur when the terminal voltage remains constant during discharge, then abruptly drops to zero at total depletion. This is also the ideal charge and discharge terminal voltage characteristics of a storage battery if such a perfect nonlinear capacitor can be considered equivalent to an ideal lossless storage battery. An energy loss must take place when the storage battery departs from this nonlinear property and exhibits an equivalent linear capacitor component in which terminal voltage drops during depletion. Discharge rate is an important parameter determining this equivalent component in practical storage batteries.

Relation (8) warns us that we must eliminate as many stages of intermediate storage as possible if we wish to make maximum use of energy from any finite source. If we cannot eliminate a stage of intermediate storage then we should avoid drawing a significant fraction of the total stored energy from that stage prior to replenishment from its source of energy. We must avoid what in engineering terminology is called deep cycling.

Two examples of the economic application of these general energy relations are as follows: first, if solar electric converters are used to augment a fossil fuel electric power network, less fuel may be consumed if all the power from the solar converters is put on the net during daylight hours, with the fossil fuel generators throttled down during daylight and taking the full load at night, than if the net draws energy from storage elements charged by the solar conversion devices. Second, less fossil fuel may be consumed in powering personnal automobiles if that fuel is burned directly in the auto, than if it were burned in a central electric power plant which is used to recharge battery powered automobiles. If battery powered automobiles are considered desirable from additional factors such as lowered pollution, then hybrid vehicles designed for shallow cycling of the storage batteries woule seem to represent an energy-efficient configuration.

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