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CHAPTER 3

INTRODUCTION

In our last discussion I presented some new energy relations that came out of the paradigm of alternatives. Starting from the assumption that nature does not care what frame of reference we use for a description, so long as we are consistent, we saw that we could define a particular complex entity that played a most important role in the concept of total scalar content. This complex entity, shown in relation (17) is what I call the energy functional.

The one thing that remains invariant when we map among all possible C-alternatives is the net scalar content, which appears as the integral of the square of the magnitude of the energy functional, taken over the whole frame of reference. Conservation of energy can then be interpreted as another way of saying conservation of this scalar content.

What broke things loose was the application of a theorem due to E.C.Titchmarsh, which demands that the two parts of the energy function, the f(s) and g(s), must be related in a very special way known as the Hilbert transform. The necessary and sufficient condition for the conservation of net scalar content is that f and g must be related in this way.

It turns out that this fact penetrates, like a well directed arrow, straight to important properties which we must find in audio circuits. It also leads to the prediction of new characteristics.

ENERGY

I set about playing with this discovery like a child with a new toy. All manner of things not only dropped out, but are still falling out of the ongoing analysis of this deceptively simple energy functional.

For example, because of the nature of the geometry of the Lebesgue square integrable space that I used to define h(s), the components of h(s) obey the multi-dimensional equivalent of the Pythagorean Theorem. (Incidentally, This space is known as a Hilbert space, having nothing to do with the Hilbert transform, although both concepts are named after the famous mathematician David Hilbert. Ironically, this paradigm brings both of these concepts together).

This means that, for each coordinate s, E(s) MUST be expressed as two parts, and these two parts must have the form which is shown in relations (18). This leads to the following fundamental relationships:

The necessary and sufficient condition for the conservation of total energy in a linear system is the partitioning of any observation of that energy into two components related through the Hilbert transform.

A "real" observable f(s) whose square is proportional to some defined energy density V(s), yields half the total energy that can be attributed to a linear process.

I defined the entity h(s) as being the energy functional, since h(s) contains all the energy density relationships. Things that fall out of this energy functional have other names. One of the things that falls directly out of the energy functional, when dealing with the time domain properties of systems such as loudspeakers, is what I have called the energy-time curve (ETC) and which we have presented in all our loudspeaker reviews.

If we look at the forms taken by h, f and g in relations (18), we cannot help but be struck by the fact that when we establish a frame of reference, with coordinates s, we will notice a certain commonality. In each coordinate, s, we will see a "real" component of energy density called V(s) which has the shape of one half times a coefficient times the square of an observable, f(s). Stop and think. How many times have we see (1/2)CV squared, or (1/2)Li squared, or (1/2)mv squared, etc. Did you ever wonder why that darn factor of one half? Or have you also ever wondered why it was that a complex representation always seemed to tag along with every measurement, even of the "real" world. How many times we have been told, in applied mathematics, that we can "solve" the equation in complex form, since it is simpler to solve in complex form, and then throw away the "imaginary" part and save the "real" part (since the "real" world was "real"). Didn't that ever bother you? It bothered the heck out of me. What was that "imaginary" structure which we had to haul along in order to get an answer, and then discard, like scaffolding which must be removed to reveal the finished structure?

Suddenly, coming at the problem from the philosophical level of the principle of alternatives, we find a whole new way of conceptualizing the situation. We are led into requiring a two-part description with every observable, with the two parts in conjugate relationship that coincides with our definition of the two-number complex.

And, oh yes, even though we will discuss this matter at some later time, we do find the Hilbert transform cropping up in all of our fundamental physics.The Hilbert transform, you see, has other names. One name is the "Cauchy-Riemann differential relations"; another name is "Cauchy's Integral"; yet another name is "Kroenig-Cramers" relations; another name is "Bode's" network relations. And for those who thought that the Hilbert transform was limited in its expression to the commonly seen integral using a reciprocal coordinate kernel, you are in for a pleasant surprise; it has many guises. One of the most fascinating, to me, is that which governs that class of operators whose dual application produces mirror symmetry. Later on in this particular discussion I will be making reference to a deep meaning of the Hilbert transform, which derives from this property, although we have not yet reached the point where we could prove it; namely, the Hilbert transform is that procedure which maps between alternative forms of energy density within a specific frame of reference. The imaginary unit is the Hilbert transform of the real unit.

OBSERVE

I just used a most interesting word. I used the word "observable". What does that mean? In terms of this paradigm the word "measure" means the same as the word "measure" as used in the mathematical definition of measure; as, in this case, Lebesgue measure. We can only "measure" that which has Lebesgue measure. The word "observable" will, in this paradigm, refer to the condition of having Lebesgue measure, and hence of being measured. The word"observe" will refer to the transfer of Lebesgue measure within a valid frame of reference. When we "measure" something (that is, when we "observe" it) we become a part of the process. It's got to be that way if we are to keep the definition of alternative, for there is no way we can stand in a part of an alternative frame of reference. We cannot observe without disturbing what we observe, even though this disturbance may be quite small. We, after all, are in the same frame of reference as that which we observe.

The definition of "observer" and "observee" might depend upon who you call what, rather than some law of nature. Does the voltmeter observe the audio oscillator, or does the audio oscillator observe the voltmeter? Or is there some system called "oscillator-voltmeter" for which there is a mutual transfer of measure. And where do you or I fit into this picture? I don't know.

"LOST" ENERGY

What about our poor villain, Snavely vonPanhandle? If you remember (AUDIO), he tried to cheat in a contest which awarded a prize to the first person who could transfer energy from a battery to a capacitor, under linear condition, wihout losing half or more of the transferred energy. He would win if he took less than two units of energy from the battery in order to end up with one unit of energy stored in the capacitor. Poor snavely, his own greed blinded him to the fact that he was the victim of a sucker bet. Nobody can win. The perpetrators of this contest pocketed the contestant's money, including Snavely's, and quietly left town, leaving everyone to wonder where the energy went. Now, let's look at the answer.

In order to show the equipartition of energy into a "stored" and "going somewhere" component, it is only necessary to remember that this is a series circuit composed of a battery, network and capacitor. The applied voltage is fixed; it is the current that changes. We don't care what the current is; we only need to remember that the voltage across the capacitor is directly related to the rate of change of this current. The energy transferred out of the battery is the net sum (integral) of the product of the battery voltage times the current. Changing the integration variable from time to voltage shows us that this transferred energy is always CV squared, regardless of what is in the box called N. Note that total energy does not have a factor of one half. When all the current has ceased, the capacitor is charged up to the battery voltage, which gives it a stored energy of one half CV squared. The ratio of energy on the capacitor to energy taken from the battery is precisely one to two, no more, no less.

This is a transitional problem in the sense that the equipartition of energy can be explained both in terms of our existing paradigm and the new paradigm of analysis. But the existing paradigm does not handle variations on this problem, while the new paradigm does.

In terms of the existing paradigm, the process of transfer of energy from a battery into such a network is performed under what is called isointensive conditions. That is, the "force", or intensive quantity (voltage, in this case), is held constant and the extensive quantity (charge and its rate of change) is unconstrained. In that case, the energy from the constrained source (battery) splits up equally into stored energy and output to unconstrained sinks. Yet it does. Really.

The new paradigm states exactly the same thing, since it includes the existing paradigm. But, in terms of the new paradigm, we can also see that the experiment was established in terms of a potential energy density frame of reference, and that the conditions of transfer of total energy from the battery were established in such a manner that there would always be an equipartition of total energy into a potential energy (that Snavely could measure) and a kinetic energy that was not available for Snavely to measure.

For linear systems, such as this one, the new energy relations (18), show that a person who uses a potential energy density frame of reference will always see V(s) with a net sum V.

Such an observer cannot see T(s) or its sum T, even though T and V are equal. Sure, Snavely might transfer out of a potential energy density frame of reference and burn his hand on a still-smoldering resistor, but he could never measure any potential.

In audio terms, if you use an oscilloscope to view the voltage on a signal line, you are utilizing a potential energy density frame of reference. Voltage as a function of time is the f(t) part of the energy functional h(t). You cannot see the g(t) part in your frame of reference; but, if you know the full f(t), you can compute g(t) by using the Hilbert transform.

By constraining the problem to use a constant voltage, which has no observable variation in terms of a potential energy frame of reference, we are assuring the equipartition of energy. On the other hand, if we allow the energy input to be expressed partly in potential and partly in kinetic, we have a new situation, which, in terms of the contemporary paradigm, is neither isointensive nor isoextensive.

The law which we derived, assures that total energy can only go into two parts. Figure ( ), which is reproduced from the 1971 paper on this matter, symbolizes the division of total energy density into two quadrature components which are related by Hilbert transform. If E is the source of energy and lies wholly along the impulse axis we have an isointensive process. If E lies wholly along the doublet axis we have an isoextensive process. If E lies elsewhere, the process is neither.

If the angle Theta remains constant during the charging process then the resultant experiment could be considered isointensive with respect to a "rotation" of axes by an amount Theta. In that case, if we observed from a potential energy density frame of reference we will see other than an equipartition of net energy. Consider the problem of figure ( ).

Same situation; switch, network and uncharged capacitor C. Only, this time, the source is a second capacitor Co which is initially charged to a voltage Vo. After the switch is closed, and C reaches a constant voltage, what is the ratio of energy stored on C to the energy transferred from Co?

It won't be one half. The source is no longer invariant in terms of the potential energy density frame of reference. And since the source frame of reference swings further away from the potential frame of reference as the capacitor charges (voltage drops as the experiment continues) the ratio will be less than one half.

I won't go through the arithmetic; it is quite straight forward. You can readily show that the ratio of stored to transferred energy is given by relation (19). The ratio is always less then one half. The highest efficiency of transfer of energy occurs when C is very much smaller than Co, which means that the voltage on Co essentially stays constant.

If the net source voltage increases during the charging process, then we can get more than half of the energy transferred to storage in C. This can occur, for example, if we use the circuit shown in figure ( ) and compute the ratio of stored energy to the energy transferred across the boundary A. In effect, part of the "half" is being expended in the source resistor R. But if we did not know there was a source resistor, we would only be aware that the source voltage was lower when we started the charging process than it was when we finished.

AUDIO IS COMPLEX

I have dwelt at length on the partitions of energy. I have a purpose in doing this. The purpose is to show that the prediction of relations (17) and (18) are very real and demonstrable are dealing in fundamental properties, not abstractions. I emphasize this point because the energy functional, from which this comes, is a COMPLEX ENTITY. cqfmn fhinkinn about audio sionals as a one stored energy to the energy transferred acros6the boundary A. In effect,ipart of the "half" is being expended in the source resistor R. But if we did not know there were a source resistor, we would only be aware that the source voltage was lower when we started the charging process than it was when we finished.

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We must stop thinking about audio signals as a one part "real" thing, and begin thinking about audio signals as two part "complex". Furthermore, we must begin thinking about the frequency domain and the time domain in the same type of terms. I personally took a lot of abuse some years ago when I insisted that the time domain response must be considered a complex entity, with an amplitude and a phase components which were related in the same manner as a comparable frequency domain characterization. Years of applying this theory have softened the blows and now the Hilbert transform is becoming common coinage in such signal analysis. It was hard in those early days to explain to engineers that the simple wiggle which they saw on an oscilloscope screen was the equivalent of a one dimensional shadow of a two dimensional phasor; that the sine wave was a side view of a constant pitch spring whose radius was constant. That, rather than being an interesting mathematical exercise, the analytic signal was a genuine manifestation of energetic exchange. Well, it is.

In future articles, I will go into greater detail on the physical significance of this complex representation. But next, we will return to basics and show that we are truly dealing with a new paradigm.

END OF CHAPTER 3