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CHAPTER 2

INTRODUCTION

For a long time I have been concerned with the fact that what we "measure" does not always correlate with what we "hear". Rather than sit back and blame it on the vagaries of subjective perception (after all, isn't the math always correct?) I have been engaged in trying to find a better math structure that might be capable of dealing with subjective perception as well as including our present math structure as a special case. As wild as that sounds, I believe I have gone a long way toward reaching this goal. It is not an overnight wonder; I have been working on this problem for nearly twenty years. TDS is one consequence of this work. In these brief articles I am presenting the technical basis of that newer math structure.

In the last article I jumped in with both feet and introduced a new integral transform and showed that the Fourier transform was nothing more than a degenerate case of this transform. The new transform is the basis for TDS.

This new transform can be regarded as a key, which we can use to open doors to domains of analysis far beyond the traditional time domain and frequency domain of contemporary theory. But, unless you know how to interpret this key, you might never know how to use it, let alone know that such doors existed. In that first article I handed you the key, with the intention of demonstrating that we are dealing with something completelynew, and not some warmed over version of contemporary analysis. In this, and in the upcoming series of articles, we will learn where that key came from, how it can be used, and whether other keys might exist. Now let's go down to the most fundamental level and begin our journey by considering a new paradigm.

A PARADIGM IS NOT FOUR NICKELS

I use the term paradigm in the sense proposed by Thomas Kuhn. In a problem solving situation, we generally resort to some conceptual model. This conceptual model is the way we "think about" a matter when trying to explain that matter to ourselves, or to others. The term paradigm has been variously applied to this situation. Paradigm may refer to the particular problem solving model itself, or in some cases may refer to a collection of persons all of whom use the same problem solving model and who, taken collectively, represent a particular discipline of thought. I will use paradigm in the sense of the model itself.

It is my contention that the mathematical structure which we use for signal analysis has a paradigm at its heart. This paradigm is never discussed and is so much a part of the way we set up problems that we might never consider it to be a paradigm until it is pointed out as such.

This hidden paradigm is revealed in the procedures that we use in traditional analysis, for we always try to describe something that is complicated in terms of things that are simpler and that are expressed in the same type of terms as the more complicated thing we're trying to describe. Therein lies the paradigm, for what we are assuming is that there are other, simpler signals, EXPRESSED IN THE SAME FRAME OF REFERENCE, which when properly combined can duplicate the messy signal.

This procedure is best exemplified in audio analysis by the traditional Fourier sine and cosine series approach to what we call "harmonic analysis". If an audio signal keeps repeating its form over and over, such as a square wave, then we know that the time waveform of this repeating signal can be duplicated by adding a series of time waveforms which are made up of sine and sodine shapes of appropriate size. There is absolutely nothing wrong in this approach, and its mathematical pedigree is beyond reproach; but, without realizing it, this hidden paradigm, that it takes time waveforms to synthesize a time waveform has locked us into a "way of thinking".

PRINCIPLE OF ALTERNATIVES

Having recognized this as a paradigm, let me now suggest that we modify the paradigm in such a way as to expand the importance of "frame of reference", yet include everything which we now use in problem solving.

Suppose I presume, as my paradigm, that nature doesn't care how we choose to look at her; there is no preferred frame of reference for representing nature.

Now, what do you mean when I say there is no "preferred" frame of reference? I really mean that I assume there are a great many different ways of describing nature, that each of these ways is totally complete in its capability of characterizing nature and doesn't depend upon the existence of any other way, and that there is nothing accounted for in one way that cannot be accounted for in any of the other ways. What I am assuming is the existence of a basic principle, which I shall call a principle of alternatives. Each of the different ways of "looking" at nature is an alternative which we may use.

The name I will give to each of the independent ways of characterization is "alternative". There are all sorts of alternatives. If we categorize a bunch of alternatives under some agree upon set of conditions, call those conditions C, then that group of alternatives will be said to be "alternatives under conditions C" or, in shorter form, “C-alternatives”.

We ought to be able to find some map which can take us from one C-alternative to another C-alternative. In other words, if you "see" nature in terms of some particular frame of reference, and if I "see" nature in terms of some alternative frame of reference, and if there is some agreed upon condition that we might call a "law of nature" (that cannot depend upon any particular coordinate value), then there must be some way of converting my view of nature to your view of nature. There should be some map, m, to convert my view, f, to your view, g, under conditions C. In math symbolism,

$$M{:}f \xrightarrow{C} g.$$

This can be interpreted in audio terms. The name of the game in audio measurements ought to be: how can I measure (f) what I hear (g) and find some way (m) to explain it. Or, more to the point, if the person who measures (f) never speaks the same language (m) as the person who hears (g), then audio measurements might as well be thrown out of the window.

CONDITION C

What about the conditions which I have labelled C? One of the things we strive for, when trying to come up with what we believe are fundamental relations, is a way of deriving as many subsequent relations as we can from this fundamental set. One has a gut feel that we're not dealing with fundamental laws when we have to tack on things which we can observe and call then "laws of nature". The more "laws of nature" we have to assume in order to make things work, the less we really know about what is going on. Let's see what we can pull out of this principle of alternatives. If there are an unlimited number of alternative ways of "looking" at nature, then it is reasonable to ask whether there is any property that does not depend upon "way of looking". Is there some fundamental property that truly characterizes an event and is not changed when we move from one alternative to another? In math parlance, what is invariant under map m?

Nothing that depends upon "frame of reference" can be depended upon to remain invariant when we map among alternatives. Connectedness and continuity cannot be depended upon, nor can the number of dimensions. One by one, the nice comfortable properties of description fall as we look at more and more alternatives.

But there is one, and as far as I have been able to determine, only one, important property that stays the same - alternative to alternative - when we consider the important descriptions of nature that we may want to use. That property is the net "how much".

To see this, consider that we have developed a legitimate description, call it F, of a natural process in terms of a frame of reference having coordinates x. This is then some F as a function of x, call it F(x). Some other being, perhaps one who utilizes an eight dimensional frame of reference, also has a legitimate description. If his (or its) frame of reference has coordinates y, then that is some G(y). And yet, another being, with coordinates z has an H(z). If F(x), G(y) and H(z) all represent alternative descriptions of each other, then what process removes all x, y and z dependence from the descriptions and results in something that cannot depend upon anyone's coordinates?

A procedure that does this is one that reduces each description to the same scalar. It reduces the dimensionality to zero. It is integration. But we cannot just use our ordinary integral. We must use one that can handle really wild and pathological (by our conventional standards) descriptions. The integral should be taken with respect to Lebesgue measure. This means that relation (13) must hold, where E is a number.

ENERGY

There is a scalar entity that is conserved when all aspects of a description are considered over the complete frame of reference. Sound familiar? Sure, it's what we call energy. But rather than postulate the conservation of energy as the fundamental law, we've come up with something rather startling. As a consequence of the principle of alternatives, there must be a property that is conserved, and this property can be called energy. We derived it from the principle.

Suppose we have established a description in a valid frame of reference which uses coordinates s. We have some E(s). Since the net Lebesgue sum is a constant E we know that relation (14) must hold.

In more conventional terminology, E(s) is an expression of energy density as a function of coordinates s, whose net sum over all s must be a scalar which we call total energy, E. As it stands, this relationship is so general as to be of little utility. However, in searching for a more useful tool to use in my paradigm, I came across a relatively obscure math theorem, having nothing to do with energy, that presented an answer. If I define a new entity, call it h(s), the square of whose magnitude is equal to E(s), then I can cast relation (14) into a manageable form. Defining h(s) as the complex entity given in relation (15), we have the expression shown in relation (16).

Bingo. Since E is finite (that is, E is a number less than infinity), we have a relationship that states h(s) is square integrable, and consequently will be of a class known in mathematics as L(superscript 2). The L stands for Lebesgue and the superscript indicates squaring. I use the superscript notation. The subscript notation L(subscript) is often seen to identify this class of integral. The condition C now becomes Conservation of net Lebesgue complex square summability.

Although not apparent at this point in our discussion, the expression of relation (15) is exceedingly important. I call h(s) the energy functional of coordinates s. h(s) contains all the energy density relationships expressed in terms of coordinates s. Things that fall out of this energy functional have other names. One of the things that falls directly out of the energy functional when dealing with the time domain properties of systems, such as loudspeakers, is what I have called the energy-time curve (ETC) and which we've presented in all of our loudspeaker reviews.

The math theorem that I uncovered is due to E.C.Titchmarsh. It is rather startling. It states that the strongest of all possible math conditions demand that the f(s) and g(s) parts of any h(s) MUST be related in a very special manner. The necessary and sufficient conditions that h(s) be square integrable is that f(s) and g(s) be related by a special process known as Hilbert transform. If the frame of reference is eight dimensional, then there will be eight s coordinates and eight corresponding h's, each with its own f and g parts. Each of the f parts will be related to its corresponding g part by the Hilbert transform.

Discovering this fact was something akin to turning the light on in an otherwise dark room. It let me go from something that was so general as to be useless, to a full set of mathematics that predicted what kinds of things we should expect to find in nature when using this new paradigm.

A MYSTERY

In the next article we will explore these new energy relationships. Until then you might find it amusing to contemplate the circuit shown in figure 2 and try to unravel the mystery that might be called "The Case of the Missing Energy".

On Monday, an advertisement is printed in the local newspaper announcing a contest in which a prize will be awarded to the first person who can transfer energy from a battery to a condenser, under linear passive conditions, without losing half or more of the transferred energy. For a mere entrance fee of one dollar, the lucky winner will receive a cool thousand dollars. The contest closes on Friday.

The experimenter is supplied with the network shown in figure 2. The capacitor in connected through a black box, shown as network N, to the switch that will allow current to flow from the battery when the judge starts the contest. The experimentzer is not allowed to know what is in the box, but is told that, whatever it is, it will not have any net voltage drop when the charging current finally drops to zero, and that the box does have a dc continuity, so that the capacitor will eventually be charged to the battery voltage. The rules are simple, the experimenter must put his own network in series with N, and his own network may be anything of his choosing, so long as it contains only linear passive components, such as resistors, inductors and capacitors, and so long as it is truly in series with N and has no third terminal connected to common. The judge will start the experiment by closing the switch and will very precisely measure the exact amount of energy drawn from the battery. The switch will remain closed until the current stops, or until the current is so small that even the judge can no longer measure any energy being drained from the battery over a period of time. Then the switch will be opened and the net amount of energy in the capacitor will be measured.

One by one, various experimenters try their hand; only to discover that the best they can achieve is a loss of half the energy delivered from the battery. Somehow, half the energy always gets lost in the box, no matter how hard they try.

Our villain, Snavely vonPanhandle, breaks into the testing laboratory before his own turn and discovers that the network contains a light bulb, a resonance circuit tuned to Channel 2 complete with an electromagnetic antenna that can send the energy off to Afghanistan, and an electric motor that drives a pump which lifts water up a standpipe while current flows from the battery, and which reverts to a generator when the water drains back down and the charging current drops off. No fool, Snavely reasons that the judges have set a trap for the unwary, not to mention the innocent. So Snavely manages to appear at the laboratory before the judge arrives, and craftily drills a hole through the box, connecting a short circuit that bypasses all these energy consuming parts.

The day before, the previous contestant had managed to get half the energy from the battery into the condenser by simply letting the network charge the capacitor directly. Sure of his victory, Snavely smugly opted for the same setup, knowing that he had short circuited all these energy consuming internal items. But, so as not to raise the judge's suspicion, Snavely placed a one millihenry inductor in series with his capacitor, knowing that the inductor had no internal resistance which could dissipate energy.

Imagine Snavely's surprise when the judge announced that he, too, had only managed to get half the energy from the battery into the capacitor. Stunned, Snavely grasped the millihenry inductor. It was ice cold. Where had the energy gone?