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CHAPTER 7

INTRODUCTION

The concept of alternatives lets us see old friends in a new light. The time domain is a C-alternative to the frequency domain. We can describe a signal either in terms of its properties in a time frame of reference, or we can hop over and describe its properties in terms of the alternative frequency frame of reference. The Fourier transform is that "shuttle train" that takes us from one of these views to the other. Unfortunately, this shuttle train has one set of tracks and only stops at two stations.

The fact that the time domain and frequency domain are alternatives explains why we cannot get a mathematically precise answer to an often voiced question: "what is the frequency as a function of time?". Such a statement is nonsense, since frequency is another way of describing time, at least in the way we use those concepts in audio.

But if the time domain and the frequency domain are only two of the infinite number of possible frames of reference, then where, in our analysis, are some of the others? The point I would make is that they wait to be discovered, like flecks of gold buried in the sand. There is one of them, however, which we can describe - the delay plane.

THREE DOMAINS

I was led to the delay plane by two pressures. First, it was a put up or shut up situation. Having stated in the technical literature that time and frequency domains were only two of an infinite number, I was obligated to come up with at least a third domain; preferably one with a different dimensionality than either the time or frequency domains. The second pressure was more insidious. Since, in human perception, there seems to be a strong need to describe sound as having a joint time-like and pitch-like property, there should be a valid alternative which embodies these properties in a complete manner.

In audio signal analysis, the time domain is one dimensional; so is the frequency domain. This third domain might be two dimensional. How do you map upward from one dimension to two dimensions? There are many ways to do that. I will describe one.

Consider figure 1. I have symbolized three alternatives, the time domain, the frequency domain and the delay domain.

It is a property of the Fourier transform that each point in time maps to the entire line in frequency. Each "place" in time has an "everywhere place" in frequency. Conversely, each point in frequency maps to the entire line in time. Suppose that the TDS transform, which can map upward to the delay domain and back down to either time or frequency, maps a point to a whole line in the delay domain, and does it in the following manner: a point in time maps to a straight line in the delay space, and a point in frequency maps to another straight line which is perpendicular to the time line in the delay space. Of course, in the delay domain, a straight line is not the whole space, as it would be in the frequency domain, but is a straight line on a two-dimensional plane. Call this plane the delay plane.

This means that each value of time represents a line in the delay plane, while each value of frequency represents a line in a direction that is at right angles to this.

A TDS TRANSFORM

An appropriate TDS transform which can map a time signal, f(t) into a two dimensional signal, g(p,T), in the delay plane is given in relation 21. The terms "p" and "T" are the coordinates in the delay plane. p stands for "pitch" and T stands for "Time delay". Similarly, the corresponding TDS transform from the frequency domain to the delay plane is given by relation 22, where f(t) and F(omega) are related by the Fourier transform shown in relation 23.

The Fourier transform uses a hyperplane exponential kernel (see our earlier discussions) and is of the first power in time and frequency; hence it cannot change dimensionality. The TDS transform that can map from one dimension to two dimensional form must use a hyper-surface exponential kernel that is of second power (quadratic) in terms of the destination frame of reference.

The TDS transform, relation 21, is actually the transform and signal characteristic which is used in the existing TDS instruments, both the Model 5842 of Bruel & Kjaer and the System 10 of Tecron.

Why do I call the new space a delay plane? Because one axis, T, represents the attribute of time delay in the time domain, while the other axis, p, represents the attribute of frequency "delay" in the frequency domain. The parameter T is expressed in units of seconds, while p is expressed in units of inverse seconds.

In order to see how this might work, consider the case where the frequency signal has the form of a pure time delay, shown in relation 23. The TDS transform of this is shown in reltion 24. The value of phase (Theta) (t), that lets this transform convert to non zero values is that for which the phase (Theta)(t) is stationary for all of its parameters. We need to take the partial derivatives of this phase with respect to frequency and evaluate the conditions on the parameters p and T which allow the partial derivatives to be zero. The easiest ones to evaluate occur as frequency goes to its infinite values. This leads to the relation show in 25.

Thus, an audio network which represents a pure time delay appears as shown in relation 26. Similarly a pure tone of frequency (omega)o appears as shown in relation 27.

The most useful situation arises when the frequency description is an all pass which can be represented as shown in relation 28. In that case the response maps to a special kind of line in the delay plane which has the properties shown in relation 29.

LOOKUP FOURIER MAP

It might appear that I've taken something nice and simple, like having two domains, and complicated it by recasting the problem into three domains. It might appear that way, but not so, for what we've done is to create a path around the Fourier transform. Even though the Fourier transform is a special case of the TDS transform, we may still want to consider the time domain and frequency domain for much of our audio analysis. We now have two ways by which we can map a frequency signal into a time signal:

(1) the direct path Fourier transform

(2) TDS transform through the delay domain.

The details are covered in the AES Journal paper. It turns out (not surprisingly) that certain elementary relationships that represent energy density show up as special forms in each of the three domains. When audio networks are composed of these elementary relationships, then the path, from frequency to time domains, involves a simple LOOKUP TABLE. This means that we do not need to evaluate the Fourier transform for such systems, we only need to add up terms which we get from a table.

The energy process which we call "resonance" shows up as "poles" in the complex frequency representation of networks. These, in turn, are represented by trajectories (lines) in the delay plane, each of which has a known time domain equivalent response.

All of a sudden we have a closed form solution for energy propagation through media which are characterized as absorptive and dispersive. We now know how the frequency response shows up as a general time smear on signals which pass through that response.

TIME DELAY DISTORTION

Describing a network (such as a loudspeaker) in terms of the delay plane allows us to see how the effect of imperfect amplitude and phase response leads to a particular type of distortion of the time delay that is imparted to signals which are reproduced through those networks. This is the meaning of the time delay distortion which the theory predicted fifteen years ago (Gee, has it been that long?).

It was the discovery of the delay plane that led to the analysis of time delay distortion. As mentioned in the first article of this series, my research has often been somewhat like the trajectory of a pinball. The items which I now present in "orderly" fashion were not created in such orderly fashion. Sometimes results occurred in random order. Once I recognized that the attribute of time delay for all pass networks was represented by simple lines on the delay plane, I sought for some way of expressing general networks in terms of more elementary networks, each of which was solvable by the use of such lines in the delay plane. I found a solution, which was first presented in reference (*) and then in more complete form in reference (*).

It turns out that every network, which is expressible in terms of what are called poles and zeroes, can be expanded as a sum of parallel networks, each of which is all pass. In other words, any audio network can be duplicated in its performance by a suitable combination of parallel networks, each of which has no amplitude change with frequency but does have a defined phase shift with frequency. There is one such all pass for each pole of the original network expression and each all pass is represented by a single line on the delay plane.

Furthermore, shifting a network forward or backward in time delay, such as moving a tweeter with respect to a midrange driver, is handled by the simple expedient of shifting the corresponding delay plane lines by the appropriate amount. The delay plane enormously simplifies an important aspect of analysis that is diffficult to handle in either the time or frequency domains.

I won't reproduce the math here, since it gets a bit complicated, but it turns out that there is a property, which we can recognize in the frequency domain, that tells us something about the way in which the time domain equivalent of that signal is smeared. The clue was the delay plane expansion which showed that we could meaningfully describe time DELAY as a function of frequency -even though the math forbids us from trying to describe time as a function of frequency. The property that shows up in the frequency domain is CIRCLENESS.

CIRCLENESS

Each elementary delay plane line in the network expansion corresponds to a frequency domain expression which describes a perfect circle on the complex plot of the energy functional. If you remember our earlier discussions, I pointed out that for each coordinate there is an energy functional h(s) having the expression,

h(s) = f(s)   g(s)

where f(s) and g(s) are related by Hilbert transform. The frequency domain and time domain and delay domain are no exception to this rule.

Now, as you might expect, there are elementary forms, call them fi(s) and gi(s), which are Hilbert transform pairs, and which can be linearly combined to duplicate any f(s) and g(s) that are due to physical processes. That is,

f(s) = Al fl(s) + A2 F2(s) + …

g(s) = Al g1(s) + A2 g2(s) + …

Each fi(s) and gi(s) correspond to what I have called a generalized all pass in the frequency domain. Each pair is represented by a line on the delay plane.

Going back to our energy relationships in C-alternative, the energy density due to each fi(s) and gi(s) are added to produce the net energy density of f(s) and g(s). Since fi(s) and gi(s) obey the Pythagorean rule in Hilbert space, they can be expressed as a constant total energy density, with the fi(s) and gi(s) expressing partitioning of that constant value into "inphase" and "quadrature" terms.

Bottom line: If we plot fi(s) and gi(s) on an fg plane, we get a perfect circle. If we plot f(s) and g(s) on an fg plane we get a form composed of circles added to circles, with each circle corresponding to a particular fi(s) and gi(s).

We can see this in the impedance plot of loudspeakers. The "resistance" and "reactance" are proportional to the "inphase" and "quadrature" terms of a corresponding energy functional. What energy functional? The cause and effect functional which expresses stored and dissipated energy as a function of frequency for a constant current.

The "pigtails" and circle form that one sees in this complex plot are manifestations of that energy exchange process which we call resonance.

Each perfect circle component in the frequency domain energy functional corresponds to a defined line in the delay domain energy functional which represents time delay as a function of frequency, and this, in turn, corresponds to unique time domain energy functional that represents one component of time delay distortion.

There may not be a simple way to describe time delay distortion since it is a mathematical concept, but here goes. The signal which comes out of a network having time delay distortion is composed of an ensemble of "rubber stamp" duplicates of the original input signal, all overlapped and added together, with each duplicate having its own unique time delay as a function of frequency.

It is like an echo chamber with each reverberation component having a different frequency dependent time delay. If you have a perfect piece of wire, the output would be a single version of the input. If you have a simple resonance in the reproducing network, the output will consist of two superimposed versions of the input signal, each version having its own special warping of time delay as a function of frequency. It is a legitimate, honest to goodness smear in time. Because the time domain is one dimensional, the overlapped components cannot be individually recognized on a time domain instrument, such as an oscilloscope. The effect of this overlapped time smear, however, is quite evident as a smearing out of the net time signal. Looking at the oscilloscope, you see the net adding-together of all the individual components of the signal. You cannot see each individual component because your view is one dimensional. If you sneak over to the alternative frequency domain view, you will see a geometric property that gives a tell-tale indication of the overlapped time domain components. Each circle on the "Nyquist plot" (which is a common name for the lower dimensional version of the fg plane view) discloses the existence of a separate time domain component: one circle, one component.

If you measure the transfer function of a network, any linear network, you would see this time delay distortion in the following manner. A time response (remember, it better be complex) will show amplitude peaks (magnitude ETC) at the mean average times of energy arrival of each component with phase rates (phase ETC) corresponding to the mean average spectral distribution of those peaks. A frequency response will show a "Nyquist plot" composed of superimposed circles, with each circle corresponding to a separate time delay component. A delay response will show superimposed bell-shaped lines, with each line correponding t a separate time delay component.

As an aside, you can begin to see why, only now after considerable theory, we begin to understand what the "meaning" of phase is in the response of loudspeakers, or what the "meaning" is of all those pigtails we see on a Nyquist plot. We have been Flatlanders, living in a lower dimensional universe, who can see the effects of higher dimensional things, but could not understand them. Some of our lower dimension colleagues have managed to muddy the waters by embellishing the significance of lower dimensional attributes, such as minimum phaseness and group delay. Sure they mean something. But the vision is so much better from a higher dimensional vantage point. Trying to explain higher dimension properties in Flatland language can be extremely frustrating.

Next up, a derivation of ETC.

END OF CHAPTER 7